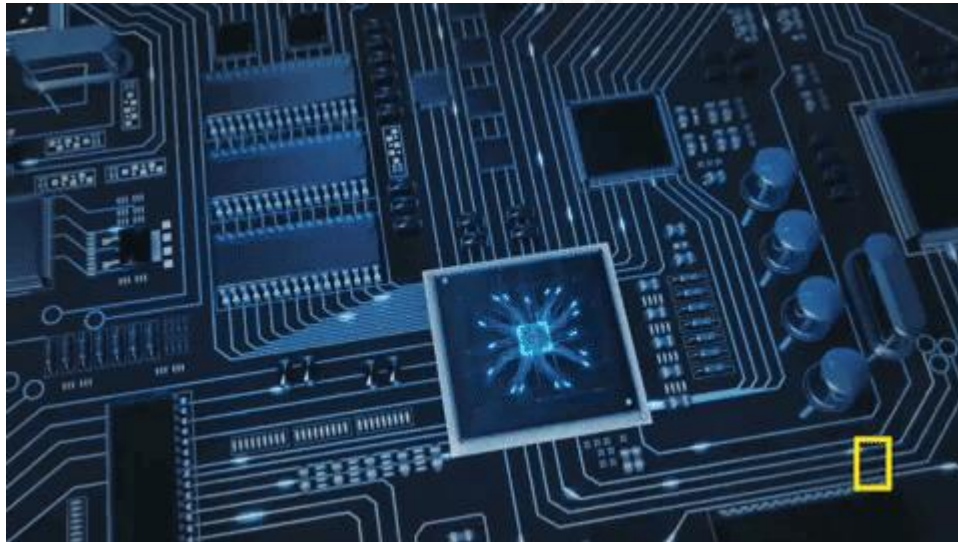


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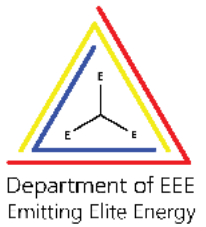
Course: Analog Electronic Circuits



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Electrical and Electronics Engineering
ATME College of Engineering, Mysuru



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Course Details

Module-3: BJT AC Analysis: Determination voltage and current gains, effect of R_L and R_S . General frequency considerations, low frequency response, miller's capacitance, high frequency response. Amplifier operation, common emitter amplifier - DC and AC analysis (base and voltage divider bias only), multistage amplifier, Cascade and cascode connections, amplifier frequency response: decibel scale, Darlington circuits, analysis and design.

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Miller's theorem

Statement:

Miller's theorem states that an **impedance Z** connected between the input and output of an amplifier with voltage gain A_v can be **replaced by two grounded impedances:**

$$Z_{in} = \frac{Z}{1 - A_v}, \quad Z_{out} = \frac{A_v Z}{1 - A_v}.$$

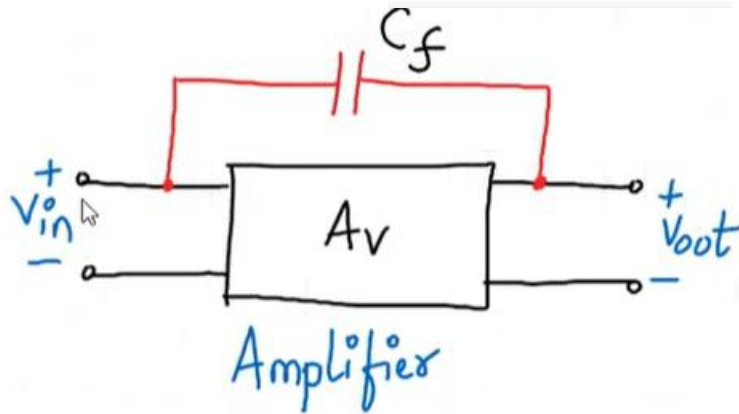
Purpose:

To **simplify circuit analysis** by converting a feedback impedance between input and output into two separate impedances to ground — one at the input and one at the output.

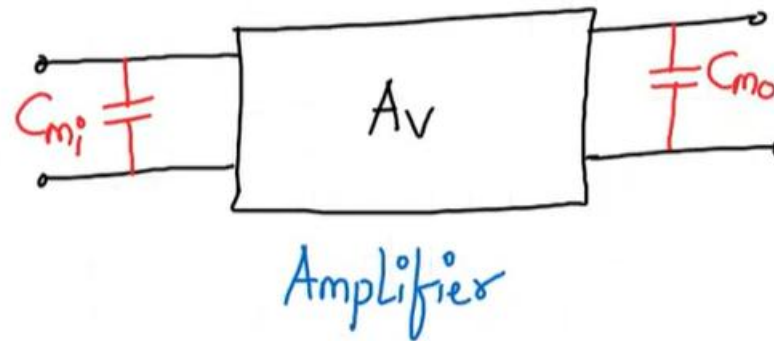
Applications:

- 1.To analyze **high-frequency response** of amplifiers.
- 2.To determine the **Miller effect capacitance** ($C_M = C(1 - A_v)$) in transistor amplifiers.
- 3.To simplify **small-signal equivalent circuits** of amplifiers, especially **common-emitter** and **inverting op-amp** stages.

Miller's theorem



C_{m_i} - Miller input capacitance
 C_{m_o} - Miller output capacitance



$$C_{m_i} = C_f (1 - A_v)$$

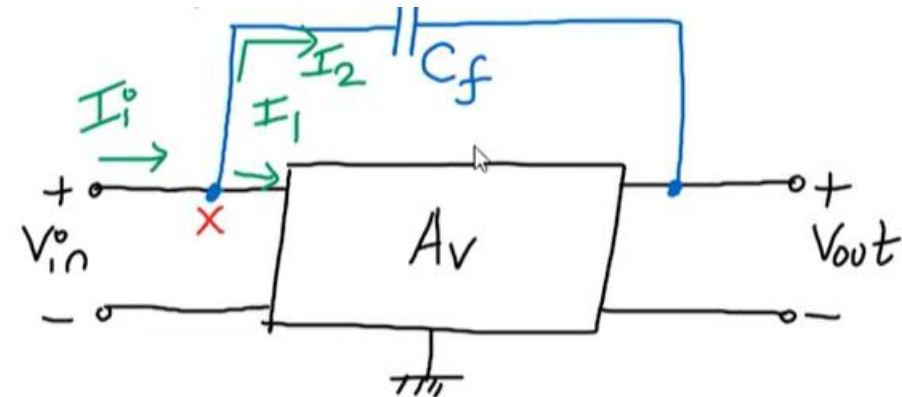
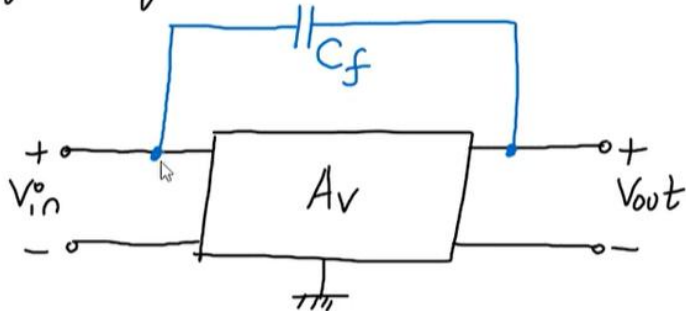
$$C_{m_o} = C_f \left(1 - \frac{1}{A_v}\right)$$

Step-1

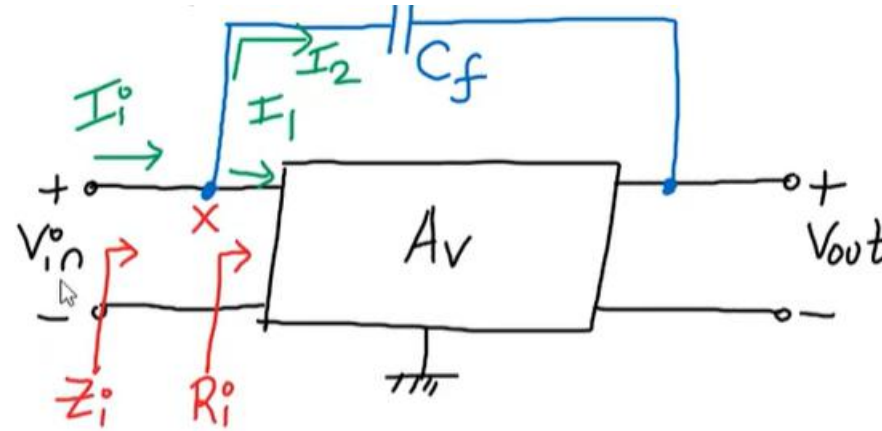
Conditions :

- ① Inverting amplifier
→ Gain A_v is negative

I] Effect of C_f on the input:



Step-2



$$\rightarrow \text{KCL at node X, } I_i = I_1 + I_2$$

$$\text{i.e. } \frac{V_{in}}{Z_i} = \frac{V_{in}}{R_i} + \frac{V_{in} - V_{out}}{X_{Cf}}$$

$$V_{out} = A_v V_{in}$$

$$\text{i.e. } \frac{V_{in}}{Z_i} = \frac{V_{in}}{R_i} + \frac{V_{in}(1 - A_v)}{X_{Cf}}$$

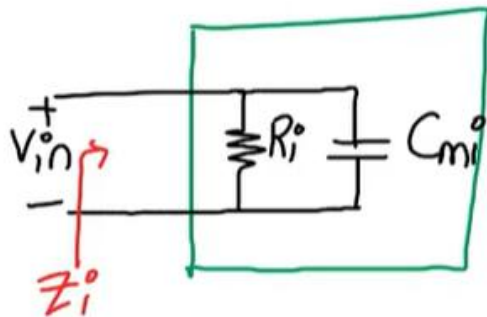
Step-3

Divide by V_{in} ,

$$\text{i.e } \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_f}/(1-A_v)}$$

$$\text{i.e } \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_{m_i}}} \dots (1)$$

$$\rightarrow X_{C_{m_i}} = \frac{X_{C_f}}{1-A_v}$$



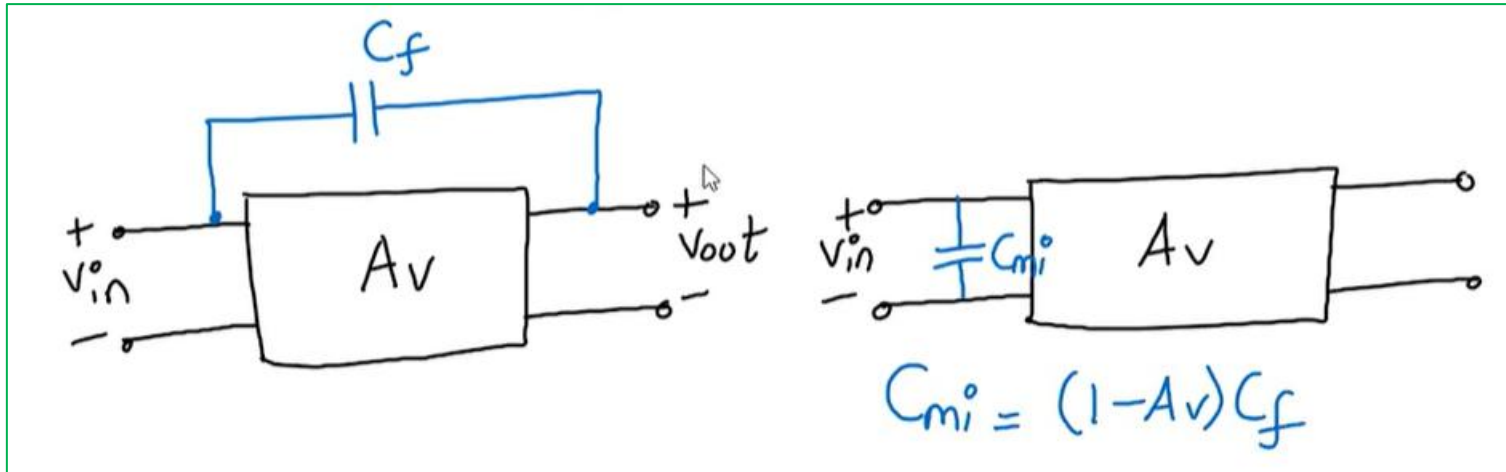
$$\rightarrow X_{C_{m_i}} = \frac{X_{C_f}}{1-A_v}$$

$$\text{i.e } \frac{1}{2\pi f C_{m_i}} = \frac{1}{2\pi f C_f (1-A_v)}$$

$$\rightarrow \boxed{C_{m_i} = C_f (1-A_v)} \dots (2)$$

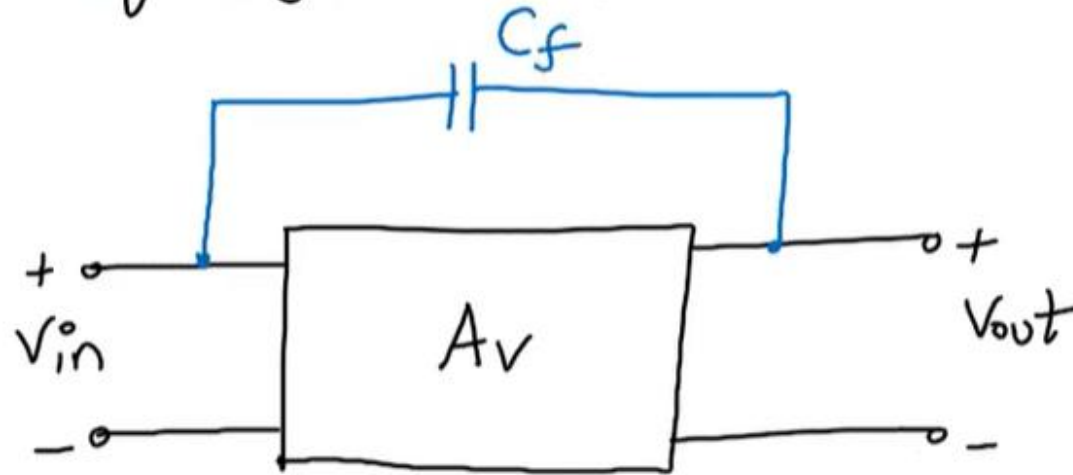
Input Miller Capacitance

Step-4

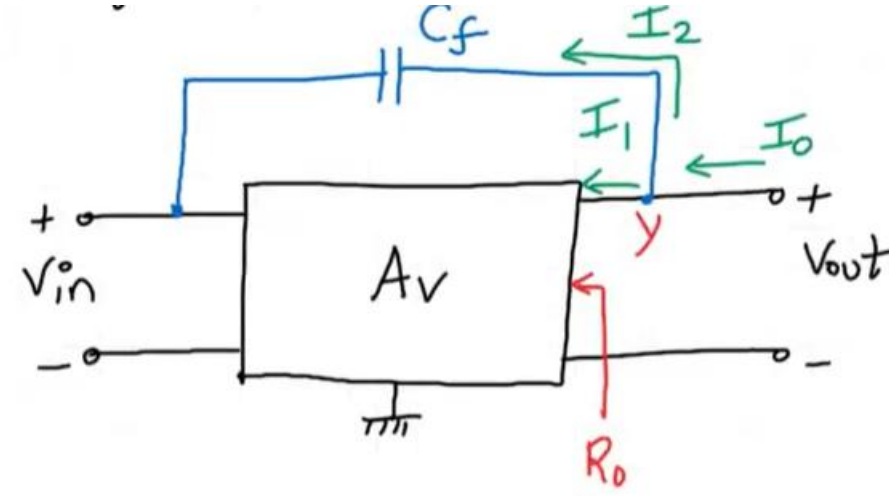
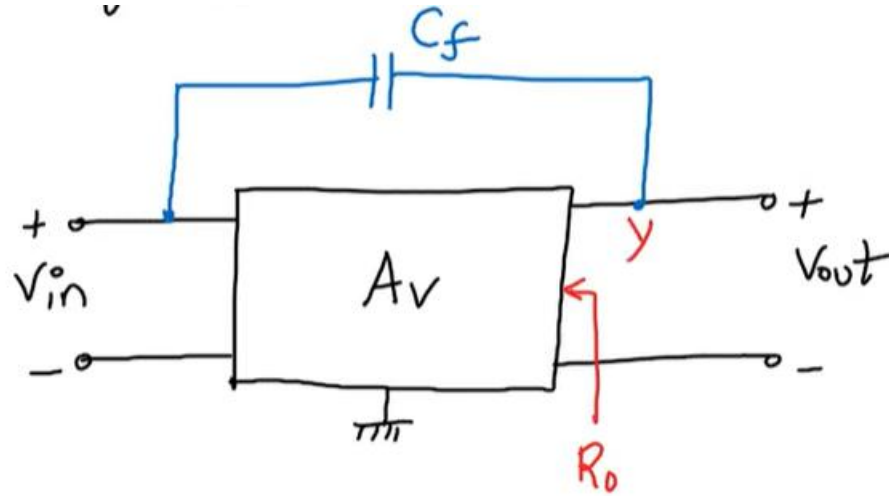


Step-1

II] Effect of C_f on Output side:



Step-2



→ KCL at node y, $I_o = I_1 + I_2$

$$\text{i.e } I_o = \frac{V_{out}}{R_o} + \frac{V_{out} - V_{in}}{X_{C_f}}$$

Step-3

R_o is usually large $\rightarrow \frac{V_{out}}{R_o} \approx 0$

$$\text{i.e } I_o \approx \frac{V_{out} - V_{in}^o}{X_{cf}}$$

$$A_v = \frac{V_{out}}{V_{in}}$$

$$V_{in}^o = \frac{V_{out}}{A_v}$$

$$\text{i.e } I_o \approx \frac{V_{out} - \frac{V_{out}}{A_v}}{X_{cf}}$$

Step-4

$$\text{i.e } I_o \approx \frac{\left(1 - \frac{1}{A_v}\right) V_{out}}{X_{Cf}}$$

$$\text{i.e } \frac{I_o}{V_{out}} \approx \frac{\left(1 - \frac{1}{A_v}\right) X_{Cf}}{X_{Cf}} \left(1 - \frac{1}{A_v}\right) = X_{Cm_0}$$

$$\text{i.e } \frac{V_{out}}{I_o} \approx \frac{X_{Cf}}{\left(1 - \frac{1}{A_v}\right)} \approx \frac{1}{2\pi f \left(1 - \frac{1}{A_v}\right) C_f}$$

Step-5

$$X_{C_f} \left(1 - \frac{1}{A_v}\right) = X_{C_{m_0}}$$

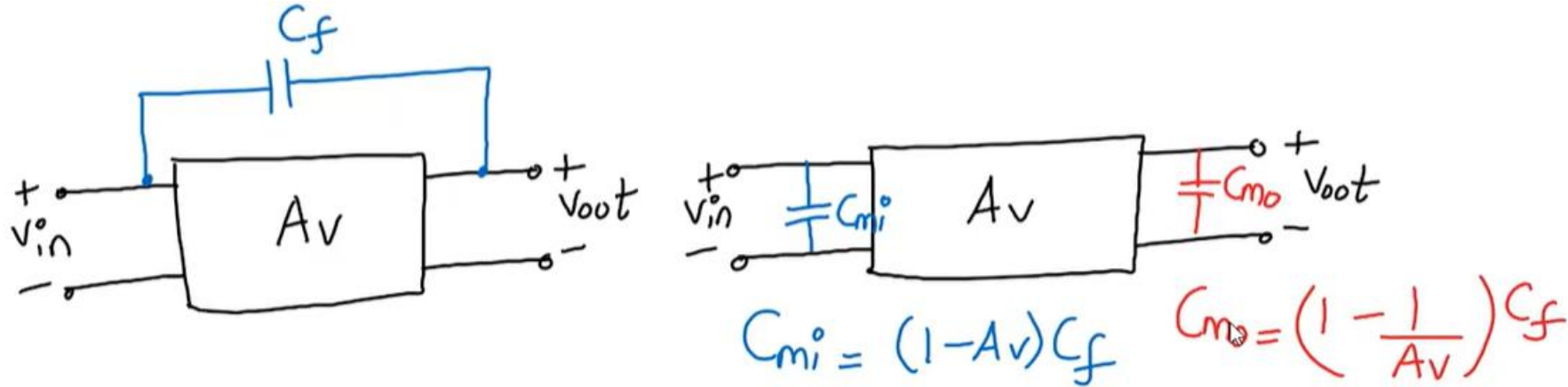
$$\rightarrow \frac{1}{2\pi f \left(1 - \frac{1}{A_v}\right) C_f} = \frac{1}{2\pi f C_{m_0}}$$

$$\rightarrow \frac{1}{2\pi f \left(1 - \frac{1}{A_v}\right) C_f} = \frac{1}{2\pi f C_{m_0}}$$

$$\rightarrow \boxed{C_{m_0} = C_f \left(1 - \frac{1}{A_v}\right)} \dots \textcircled{3}$$

$$\text{If } A_v \gg 1 \rightarrow \boxed{C_{m_0} \approx C_f}$$

Step-6



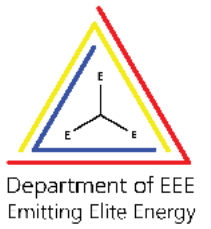
→ Inverting amplifier (CE amplifier, CS amplifier)

$$A_v \gg 1 ; C_{mi} \gg C_f$$

$$C_{mo} \approx C_f$$



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Miller effect: Increase in the capacitance at the input is called "Miller effect"



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State and prove the dual of Miller's theorem

Miller's theorem (impedance form) — statement

If an impedance Z is connected between two nodes whose small-signal voltages are related by

$$v_2 = A_v v_1,$$

then the effect of Z on the circuit can be replaced by two equivalent impedances to ground:

an input equivalent impedance

$$Z_{\text{in}} = \frac{Z}{1 - A_v}$$

connected between node-1 and ground

an output equivalent impedance

$$Z_{\text{out}} = \frac{Z}{1 - \frac{1}{A_v}},$$

connected between node-2 and ground.

Proof of Miller's theorem

Consider the impedance Z between node 1 (voltage v_1) and node 2 (voltage v_2). The voltage relation is

$$v_2 = A_v v_1$$

The current through Z (from node 1 toward node 2) is

$$i_Z = \frac{v_1 - v_2}{Z} = \frac{v_1 - A_v v_1}{Z} = \frac{(1 - A_v) v_1}{Z}.$$

Define an equivalent impedance Z_{in} from node 1 to ground that draws the same current when node-1 has voltage v_1 . That is,

$$i_Z = \frac{v_1}{Z_{in}}.$$

Equate the two expressions for i_Z

$$\frac{v_1}{Z_{\text{in}}} = \frac{(1 - A_v) v_1}{Z} \Rightarrow Z_{\text{in}} = \frac{Z}{1 - A_v}$$

Similarly, express i_Z as seen from node 2. Current through Z from node 2 toward node 1 is

$$i'_Z = \frac{v_2 - v_1}{Z} = \frac{v_2 - v_2/A_v}{Z} = \frac{\left(1 - \frac{1}{A_v}\right) v_2}{Z}$$

Define Z_{out} such that $i'_Z = \frac{v_2}{Z_{\text{out}}}$ $\Rightarrow$ $\frac{v_2}{Z_{\text{out}}} = \frac{\left(1 - \frac{1}{A_v}\right) v_2}{Z}$ $\Rightarrow$ $Z_{\text{out}} = \frac{Z}{1 - \frac{1}{A_v}}$

Electrical duality swaps:

- voltages $\leftrightarrow$ currents,
- impedances $Z \leftrightarrow$ admittances Y (where $Y = 1/Z$),
- series connections $\leftrightarrow$ parallel connections,
- voltage gain $A_v \leftrightarrow$ current gain K (or α).

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Dual statement:

If an **admittance** Y is connected between two nodes of a network whose **currents** are related by a current gain K (i.e. the controlling relationship is $i_2 = K i_1$ in the dual problem), then the admittance Y can be replaced by two **equivalent admittances to ground** (i.e. two shunt elements) given by the dual formulas (obtained by the same algebraic replacement rules).

Written compactly (by direct analogy to the impedance formulas) the dual pair becomes:

$$Y_{\text{in}} = \frac{Y}{1 - K}, \quad Y_{\text{out}} = \frac{Y}{1 - \frac{1}{K}}.$$

(Here K is the appropriate current-gain in the dual/current representation.)

How to justify the dual: take the impedance proof above, replace:

$$v \longrightarrow i, \quad Z \longrightarrow Y, \quad A_v \longrightarrow K,$$

and reinterpret series relations as parallel ones. The algebraic steps are identical after these substitutions, so the same form of relations holds for admittances under current-gain conditions.

Transistor frequency response:

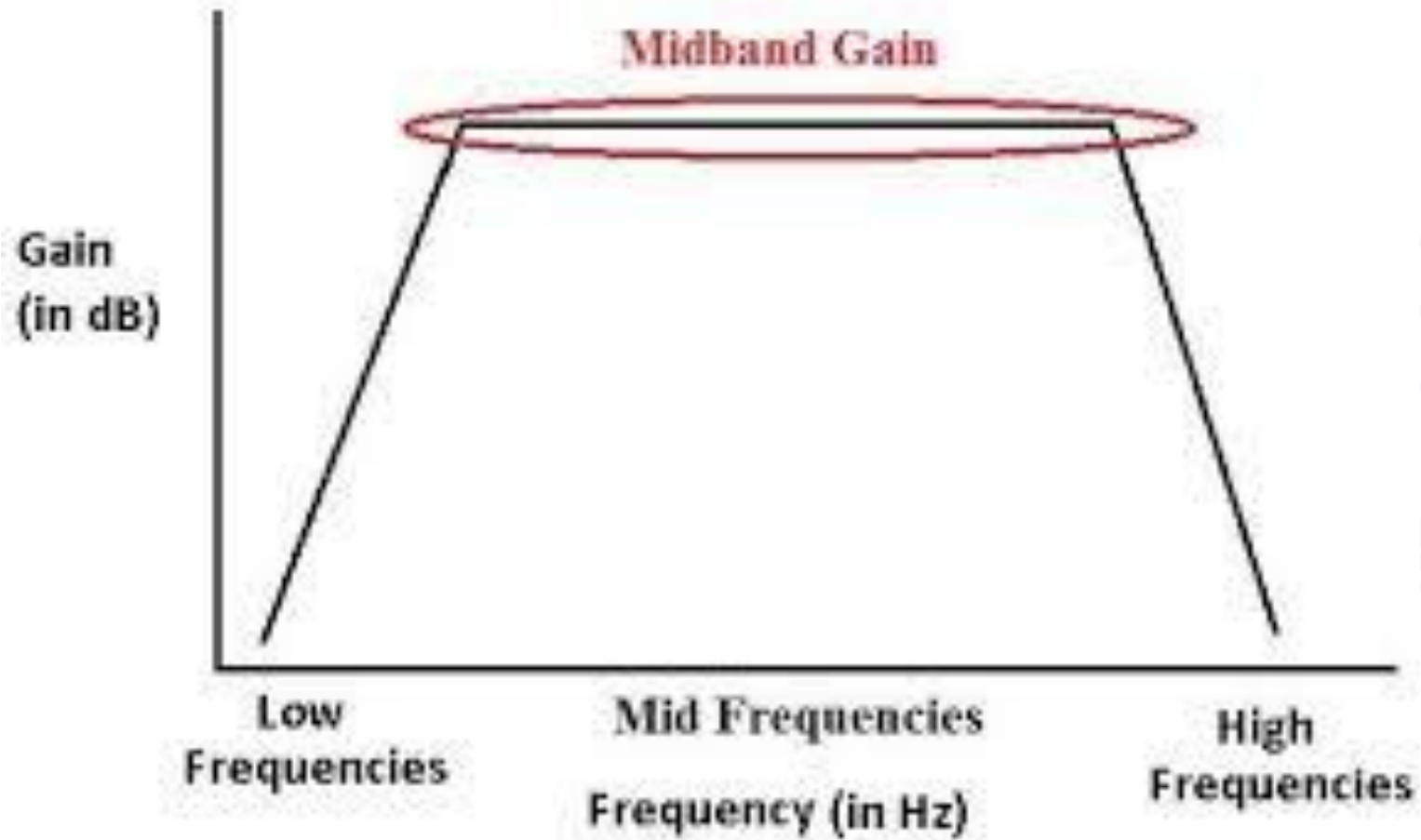
Voltage Gain (A_v)

If an amplifier increases a signal:

$$A_v = \frac{\text{Output Voltage}}{\text{Input Voltage}}$$

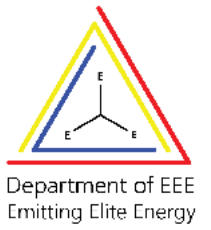
Example:

If input is 1V and output is 100V $\rightarrow$ gain is 100.



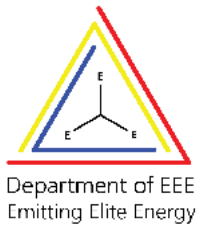


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Transistor frequency response:

General frequency considerations:

Concept of frequency response:

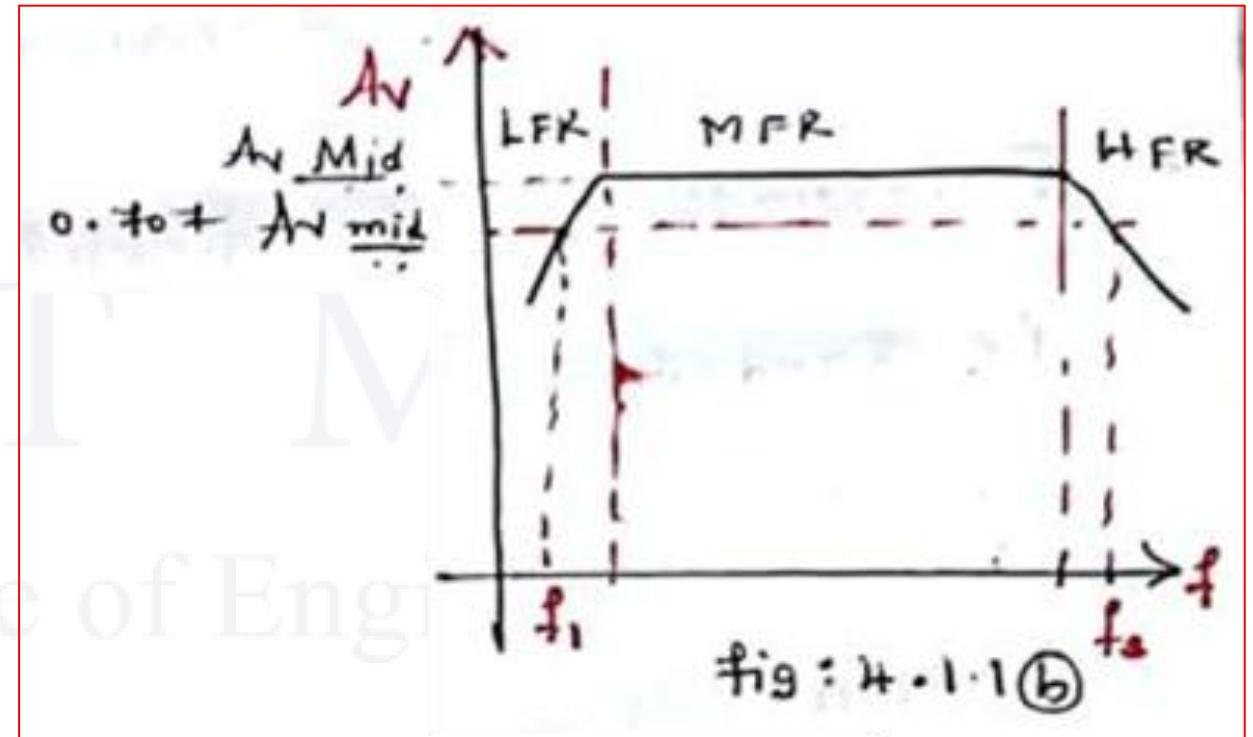
An amplifier should ideally provide the same **amplification for all frequencies.**

The response to which this happens is usually indicated by a curve known as **frequency response curve of the amplifier.**

To plot this curve, input voltage to the amplifier is kept constant, and the **frequency of input signal is continuously varied**.

The output voltage at each value of input signal's frequency is noted and **gain of the amplifier** is calculated.

The ratio of V_o / V_i (or the voltage gain of the amplifier) is then plotted against frequency.



This curve is a plot of the **voltage gain** of an amplifier against the **frequency** of the input signal.

What are 3 dB Frequencies (f_1 and f_2)?

As you increase or decrease frequency:

Middle frequencies → **gain is maximum** (flat region)

At very low or very high frequencies → **gain decreases**

The points where gain **drops by 3 dB** are called:

- f_1 = lower 3 dB frequency
- f_2 = upper 3 dB frequency

Why is 3 dB important?

Because 3 dB drop means power becomes half.

So, f_1 and f_2 are **half-power points**.

What is 3 dB Bandwidth?

Bandwidth = $f_2 - f_1$

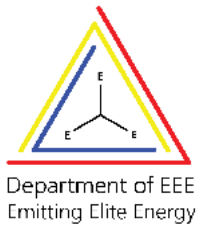
This tells us how wide the useful frequency range of amplifier is.

More bandwidth = amplifier works correctly over more frequencies

Less bandwidth = limited frequency performance

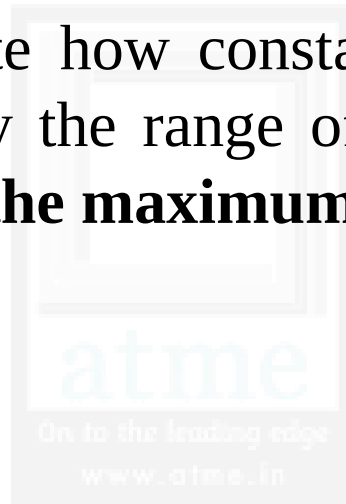


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Cut-off Frequency and Bandwidth

To indicate how constant an amplifier's voltage gain is with frequency variation, we may specify the range of frequencies over which the gain does not vary more than **+ or – 1 dB of the maximum gain at some reference mid-frequency**.



- The frequency response is nearly **ideal** over a wide range of **mid-band frequencies**.
 - At low and high frequency ends, the **gain deviates from ideal characteristics**.
 - The decrease in voltage gain with frequency is called **roll-off**.
- The two frequencies at which voltage gain starts decreasing below +1 dB are indicated by f_1 & f_2
- These are called the **lower cut-off frequency** and **upper cut-off frequency** respectively.

• **Bandwidth** of the amplifier is defined as the difference between f_2 & f_1 .

i.e., Bandwidth of the amplifier = $f_2 - f_1$.

The frequency f_2 lies in high frequency region, while the frequency f_1 lies in low frequency region.

These 2 frequencies are also referred to as **half-power frequencies**, since gain (G_v) at f_1 or f_2 drops to **70.7%** of maximum value.

Example:

The voltage amplifier has voltage gain = 200 at cut-off frequency.

Find the maximum voltage gain.

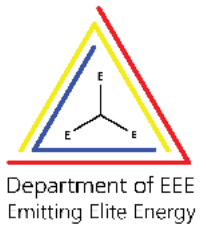
Solution:

- Maximum voltage gain = Gain at $\omega = \text{cut-off} \times \sqrt{2}$
 $= 200 \times \sqrt{2}$

Maximum Vtg gain = 282.84



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Gain in Decibels (dB)

Instead of writing large numbers like gain = 100, 500, 1000...

Engineers prefer **decibel scale**, because:

- ✓ Easy to compare
- ✓ Shows how strong the amplification is
- ✓ Useful in frequency response graphs

$$V_{tg} \text{ gain in dB} = 20 \log(A_v)$$

The Decibel Unit

Usually, the voltage gain of the amplifier is represented in decibels (dB).

It is given by:

$$\text{Vtg gain in dB} = 20 \log(A_v)$$

For example, assume that mid-frequency gain of certain amplifier is 100.

Then voltage gain in dB:

$$\text{Vtg gain in dB} = 20 \log 100 = 40 \text{ dB}$$

$$\text{At } f_1 \text{ \& } f_2, A_v = \frac{100}{\sqrt{2}} = 70.7 \text{ dB}$$

- Vtg gain in dB at f_1 = Vtg gain in dB at $f_2 = 20 \log (A_v / \sqrt{2})$
= 3 dB

It shows that the Vtg gain at f_1 and f_2 is less than 3 dB of the maximum Vtg gain.

Due to this, frequencies f_1 and f_2 are also called **3 dB frequencies**.

- The bandwidth ($f_2 - f_1$) is known as **3 dB bandwidth**.
- f_1 is known as **lower 3 dB frequency** and f_2 is known as **upper 3 dB frequency**.



Significance of Octave and Decade

- The octave and decade are the measures of change in frequency.

A ten-time change in frequency is called a **decade**.

- On the other hand, an **octave** corresponds to a doubling ($\times 2$) or halving ($\div 2$) of the frequency.

•For example:

An increase in frequency from 100 Hz to 200 Hz is an octave.

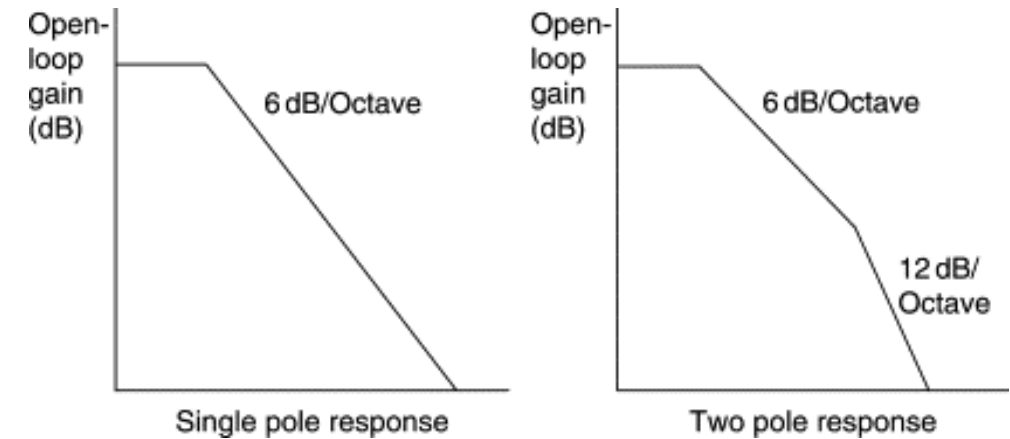
Likewise, a decrease in frequency from 100 kHz to 50 kHz is also an octave.

•At lower and higher frequencies, the decrease in the gain of amplifier is often indicated in terms of **dB per decade** or **dB per octave**.

If the attenuation in the gain is 20 dB per decade, then it is indicated by line having slope of **± 20 dB/decade** — this is illustrated in Fig.

•A slope of -20 dB/decade is approximately equivalent to -6 dB/octave.

A slope of -40 dB/decade is approximately equivalent to -12 dB/octave, and so on.



Midband Gain:

- In the midband, the voltage gain of the amplifier is approximately maximum.

It is represented as midband gain (A_{mid}).

- The voltage gain of the amplifier outside the midband is approximately given as:

$$A = \frac{A_{mid}}{\sqrt{1 + (f_1/f)^2} \sqrt{1 + (f/f_2)^2}}$$

In the midband, $f_1/f \approx 0$ and $f/f_2 \approx 0$

Midband: $A = A_{mid}$

Case-1: Below the midband

When $f \ll f_2$, the equation becomes:

$$A = \frac{A_{mid}}{\sqrt{1 + (f_1/f)^2}}$$

Case-2: Above the midband

When $f_1/f \approx 0$, the equation becomes:

$$A = \frac{A_{mid}}{(1 + (f/f_2)^2)}$$

Example

For an amplifier, midband gain = 100

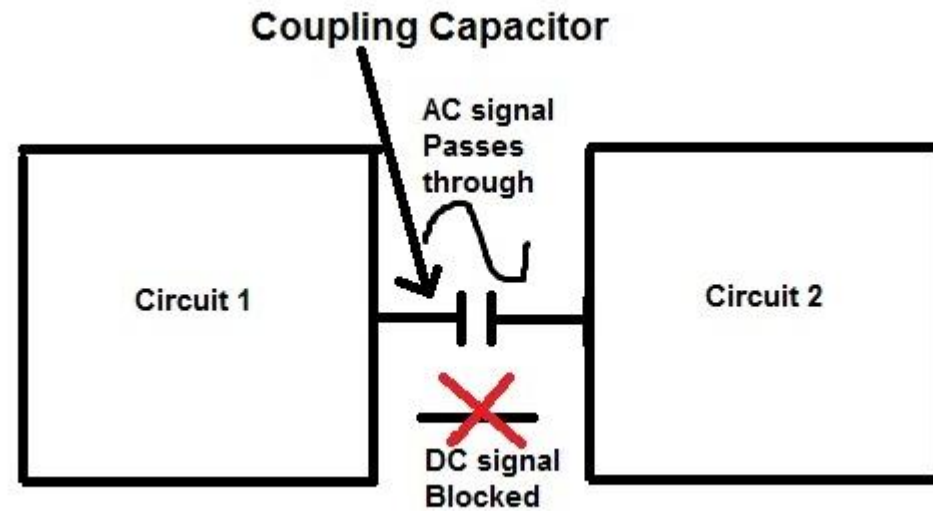
Lower cut-off frequency = 1 kHz

Find the gain of an amplifier at frequency $f = 20f_c = 20 \times 1000 = 20000 \text{ Hz}$.

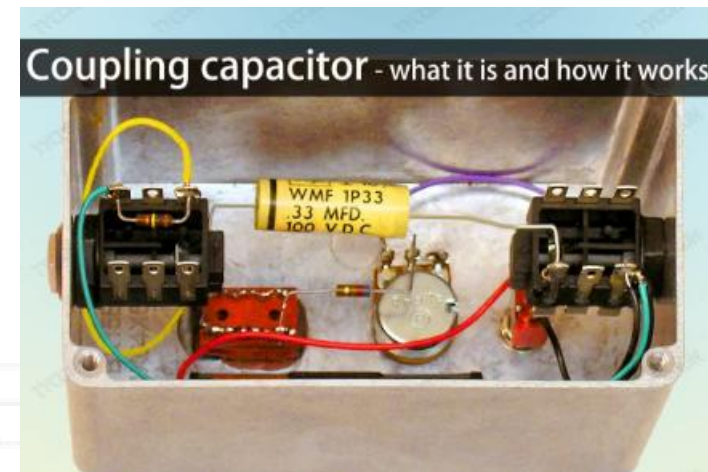
Solution:

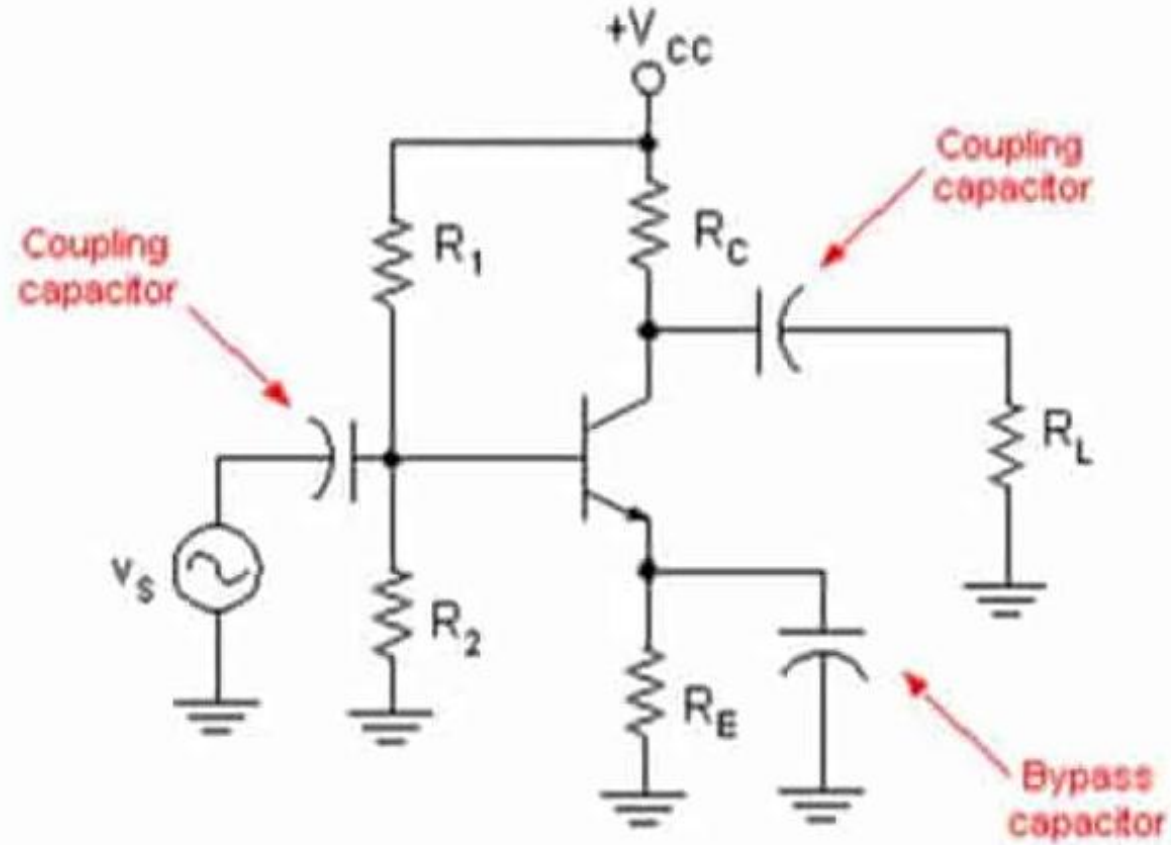
Below midband:

$$\begin{aligned} A &= \frac{A_{mid}}{\sqrt{1 + (f_c/f)^2}} = \frac{100}{\sqrt{1 + (1000/20000)^2}} \\ &= \frac{100}{\sqrt{1 + (0.05)^2}} = \frac{100}{\sqrt{1.0025}} \approx 99.75 \end{aligned}$$



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The **coupling capacitor** couples and isolates the two systems of **strong current** and **weak current** through the capacitor, providing a high-frequency signal path to prevent low-frequency current from entering the weak current system, and ensure safety.

<https://www.tycorun.com/blogs/news/coupling-capacitor?srsId=AfmBOoprDJ9dSa5yfygALz-wtrDodCYylsh8Jj8AEo3rQ9PVeZXD51go>

Effect of Coupling Capacitors:

A coupling capacitor is used in amplifier circuits to **pass AC signals** but **block DC**

A capacitor's reactance is:

$$X_C = \frac{1}{2\pi fC}$$

Case-1: When **frequency (f)** is **high**, X_C becomes **very small**

- Capacitor acts like a **short circuit** → Signal passes easily
- Amplifier gain remains normal

Case-2: When **frequency is low**, **f decreases** → **X_C becomes large**

1. Capacitor starts **blocking** the AC signal
2. Less signal reaches the next stage
3. Amplifier gain drops
4. Output voltage becomes small

Example

- At **1 kHz**, capacitor behaves like a wire → full signal passes → good amplification
- At **10 Hz**, capacitor blocks most of the signal → very little output → poor amplification

What is CE?

CE = Common Emitter

- Used in **Bipolar Junction Transistor (BJT)** amplifiers
- The **Emitter** terminal is *common* (shared) between input and output (usually connected to ground through a resistor)
- Mostly used for **voltage amplification**

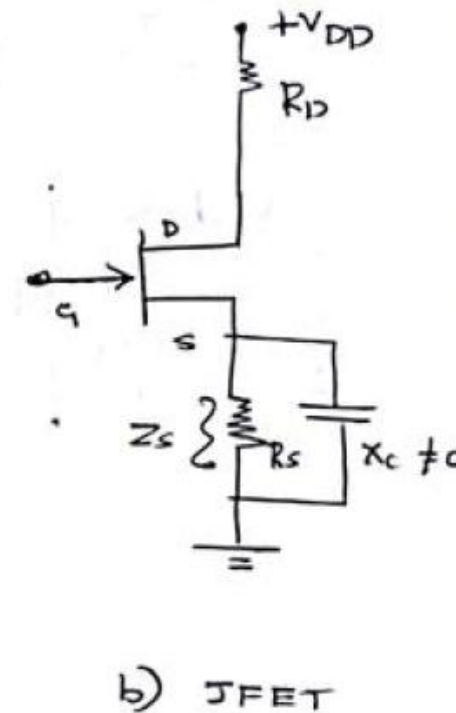
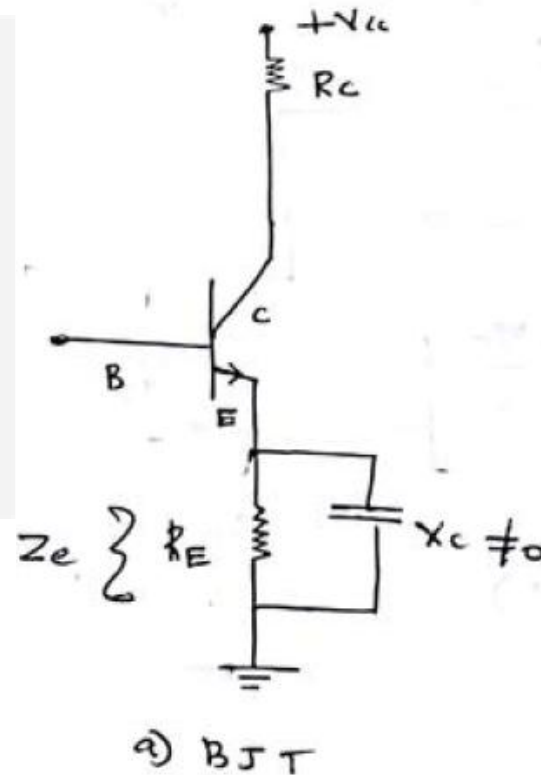
What is CS?

CS = Common Source

- Used in **FET (Field Effect Transistor)** amplifiers
- The **Source** terminal is *common* between input and output
- Works similar to CE amplifier in BJTs

Effect of Bypass Capacitance

A **bypass capacitor** (usually CE in BJT, CS in FET) is connected across the emitter resistor to **improve gain**.



What happens at different frequencies?

At high or mid frequencies

Reactance $X_C = \frac{1}{2\pi fC}$ becomes very small

CE acts like a **short circuit**

The emitter is at **AC ground**

No AC signal voltage drops across R_E

✓ Gain becomes high

At low frequencies

Frequency is low $\rightarrow X_C$ becomes *large*

Bypass capacitor **no longer behaves like a short**

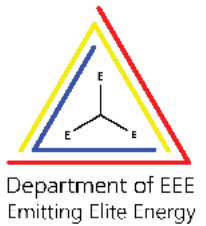
AC signal now drops across R_E

This creates negative feedback

✗ Gain reduces



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At **low frequencies**, the bypass capacitor does not act as a short circuit because its reactance increases.

Therefore, the emitter is no longer at AC ground and the input signal develops voltage across R_E , producing negative feedback. **This reduces the voltage gain of the amplifier.**

Effect of Internal Transistor Capacitances

At **high frequencies**, the external capacitors in the amplifier (coupling and bypass capacitors) behave like short circuits.

So, they do **not** affect the amplifier gain at high frequency.

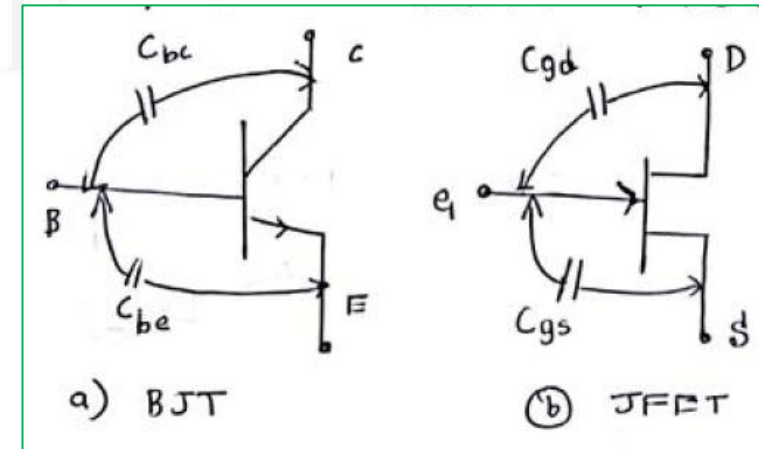
- But inside every transistor, there are **internal capacitances** called **junction capacitances**.

For BJT:

- C_{be} → Base–Emitter capacitance
- C_{bc} → Base–Collector capacitance

For JFET:

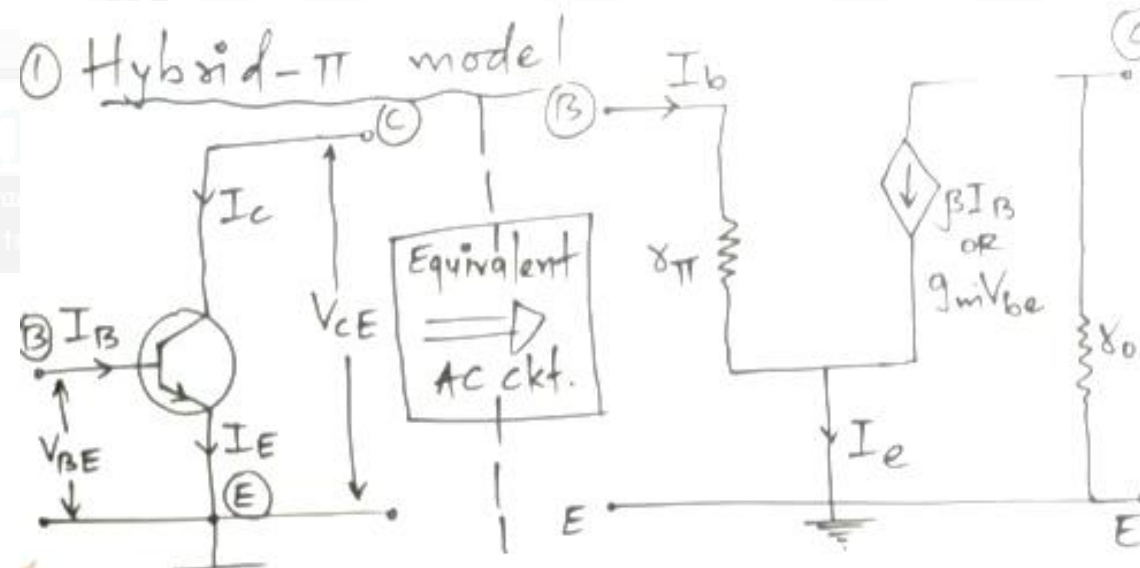
- C_{gs} → Gate–Source capacitance
- C_{gd} → Gate–Drain capacitance



1. Hybrid- π Model of Transistor

The **hybrid- π model** is a **small-signal high-frequency model** of a BJT.

It includes internal transistor resistances and capacitances that affect gain at high frequencies.



1) Small Signal Resistance (r_π)

$$r_\pi = \frac{V_{be}}{I_b}$$

2) Output Resistance (r_o)

$$r_o = \frac{V_A}{I_S}$$

Higher collector current $\rightarrow$ smaller r_o

Lower current $\rightarrow$ larger r_o

Small-Signal Resistance (r_π)

- This is the **AC input resistance** seen between **base and emitter** of a BJT.

- It tells how much the base-emitter junction resists small AC signals.

Larger base current $\rightarrow$ smaller r_π

Smaller base current $\rightarrow$ larger r_π

This is the **AC resistance** seen looking into the **collector**.

Where:

- V_A = Early voltage
- I_C = Collector current

3) g_m :

$$R = \frac{V}{I}$$

$$g_m = \frac{I}{V}$$

$$g_m = \frac{I_c}{V_{be}}$$

Higher $I_C \rightarrow$ higher $g_m \rightarrow$ higher gain

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Transconductance (g_m)

- It tells how much **collector current** changes for a small change in **base-emitter voltage**.
- It represents the "strength" of the transistor as an amplifier.

CE short circuit current gain using hybrid- π model

In common-emitter (CE) configuration, the **short-circuit current gain** tells us how much **collector current** flows for a given **base current** when the **output is shorted** (collector tied to emitter $\Rightarrow V_{CE}=0$ v)

Step-1: To analyze this at high frequencies, we use the **hybrid- π model**:

- r_{π} and C_{π} between base–emitter
- C_{μ} between base–collector
- controlled current source $g_m v_{\pi}$ from collector to emitter

Step-2: When the output is shorted, the collector goes to AC ground.

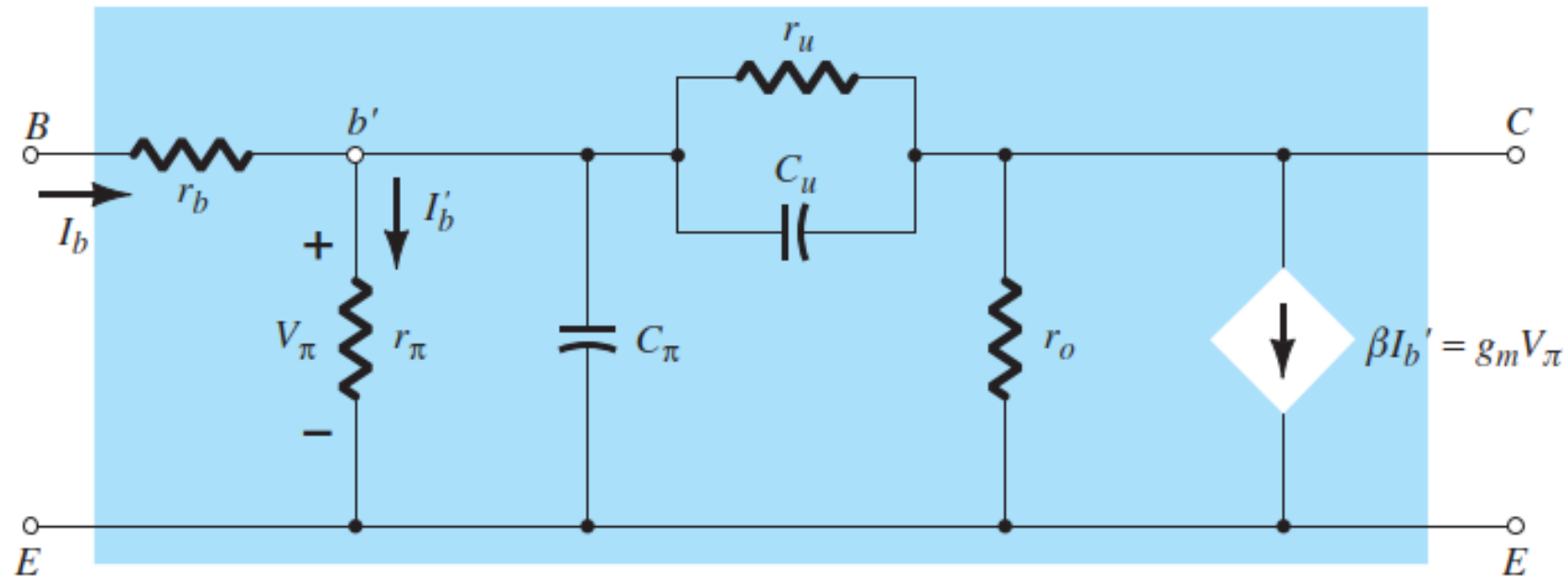
Therefore:

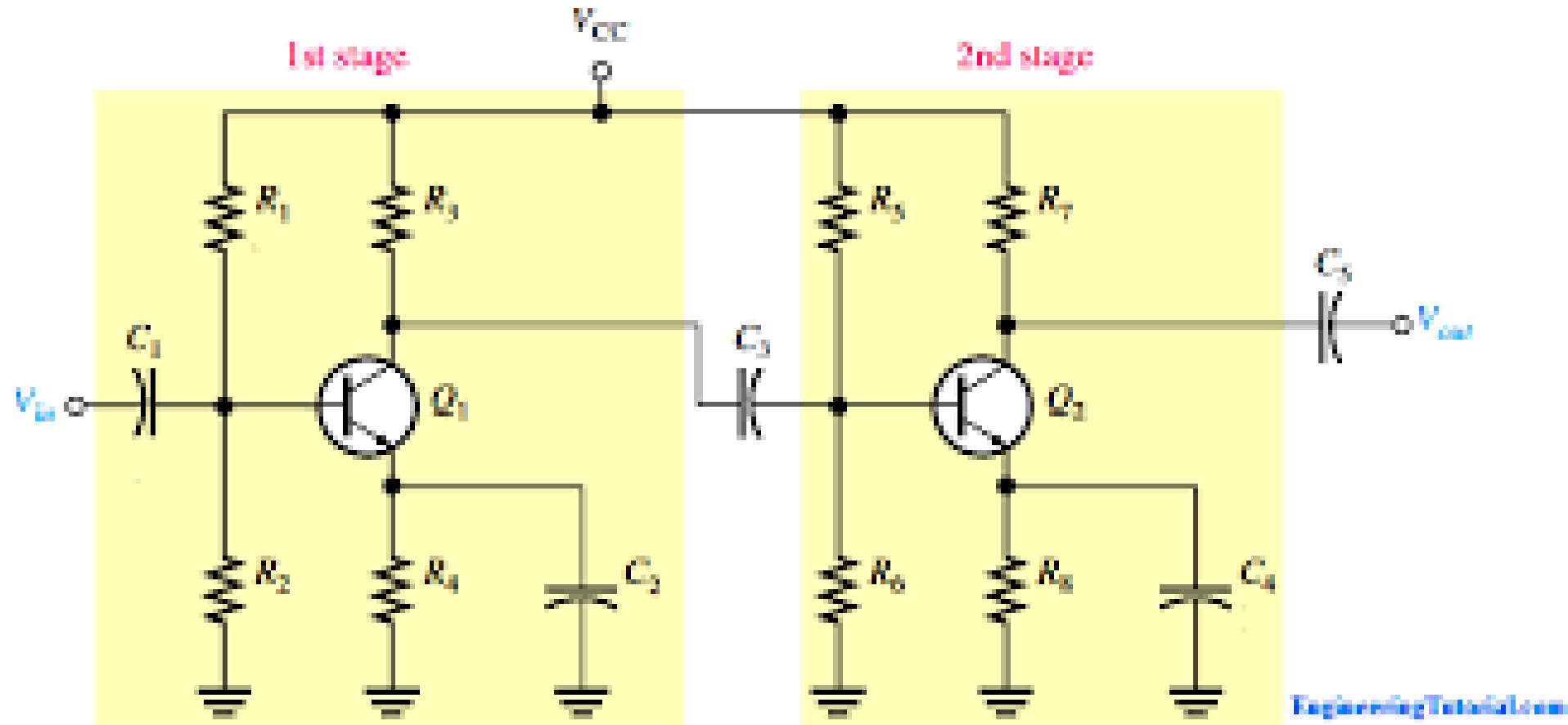
- C_μ becomes a capacitor from base to ground
- C_π also goes to ground
- collector resistance r_o has no voltage across it $\rightarrow$ ignored

Step-3: Then we write KCL (current equations) at the base and at the collector to find:

$$\beta_{sc}(s) = \frac{i_c}{i_b} = \frac{g_m + sC_\mu}{g_\pi + s(C_\pi + C_\mu)}$$

This shows how current gain decreases with frequency due to capacitances.





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Multistage Frequency Effects

When an amplifier has **more than one stage** (e.g., $CE \rightarrow CE$, $CE \rightarrow CB$, $CE \rightarrow CC$), the **overall frequency response** is not simply the response of a single stage.

Multiple stages introduce **additional poles, zeros, and loading** effects that change gain at high and low frequencies.

1. Loading Effect between Stages

Each stage's **input impedance** loads the **previous stage's output**.

- At **mid-frequency**, loading reduces gain slightly.
- At **high-frequency**, capacitive reactances reduce input/output resistances → more loss.

Example:

- Stage-1 output resistance + Stage-2 input capacitance creates another RC low-pass effect → **extra pole**.

2. Coupling and Bypass Capacitors

In multistage amplifiers:

- Coupling capacitors C_C appear between stages
- Bypass capacitors C_E at emitters

Each capacitor with its RC network generates a **low-frequency pole**.

If there are 3 coupling capacitors, total low-frequency response may have 3 **poles** → **slower low-frequency roll-off**.

3. Miller Effect Increases Capacitance

In common-emitter stages, the **Miller effect** multiplies

$$C_M = C_\mu(1 + A_v)$$

More stages → more Miller capacitances accumulated → **bandwidth decreases.**

What is a POLE?

A **pole** is a frequency where the system's gain starts to decrease.

- It **reduces gain** at higher frequency.
- It adds **-20 dB/decade** slope per pole.
- It also adds **-90° phase shift** (eventually).
- ☐ **Poles make the system slower.**

Example:

A resistor + capacitor (RC low-pass) always creates a **pole**.

What is a ZERO?

A **zero** is a frequency where the system's gain starts to increase or stops decreasing.

- It can **boost gain** or flatten the drop.
- It adds **+20 dB/decade** slope per zero.
- It adds **+90° phase shift** (eventually).
- ☐ **Zeros add a bump or boost in response.**

Example:

A capacitor that bypasses a resistor (like in an emitter bypass) creates a **zero**.

4. Inter-stage Capacitances Add Poles and Zeros

Capacitances connect nodes between stages:

- Base-collector capacitances
- Wiring, PCB parasitics
- Device parasitics

They create **poles and sometimes zeros** → affects phase shift and stability.

Example:

$$f_p = \frac{1}{2\pi RC}$$

If **C increases due to multiple stages** → **fp reduces** → **bandwidth reduces**.

5. Dominant and Non-dominant Poles

Each stage has its own pole:
 $p_1, p_2, p_3, \dots$

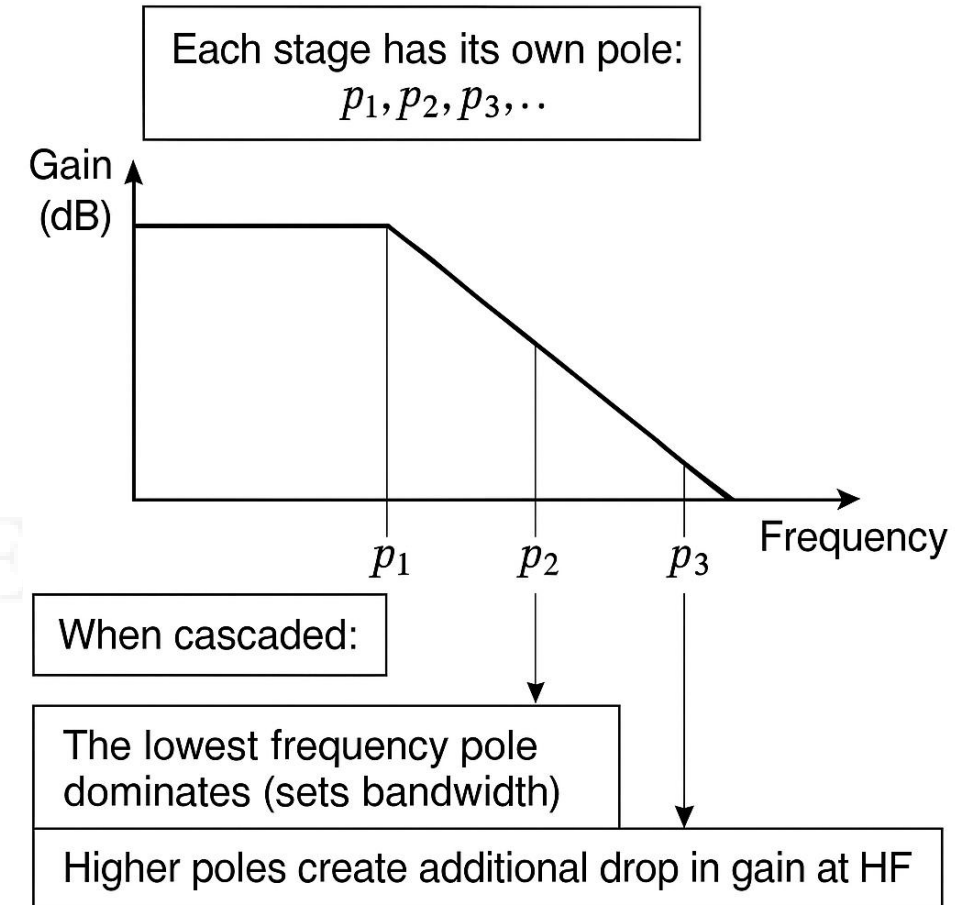
When cascaded:

- The **lowest frequency pole** dominates (sets bandwidth),
- Higher poles create additional drop in gain at HF.

Overall gain roll-off:

- Single stage $\rightarrow -20$ dB/decade
- Two stages $\rightarrow -40$ dB/decade
- Three stages $\rightarrow -60$ dB/decade

So high-frequency drop becomes **steeper**



6. Low-Frequency Gain Drops Earlier

Because each stage adds coupling capacitors, low-frequency cutoff shifts **upward**:

$$f_L \approx \text{max of all pole frequencies}$$

So, if single stage had $f_L = 20 \text{ Hz}$,
two-stage may rise to 60–100 Hz.

7. High-Frequency Bandwidth Shrinks

Because more poles accumulate, high-frequency cutoff shifts **downward**:

$$f_H \approx \text{lowest of high-frequency poles}$$

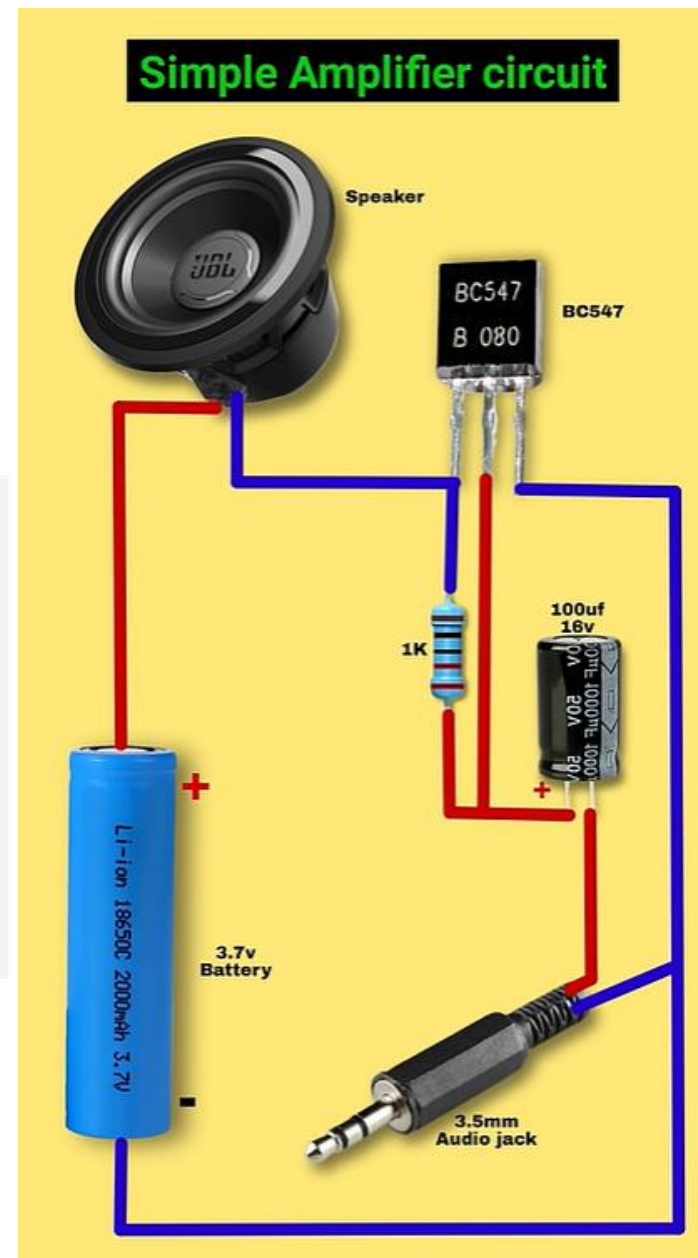
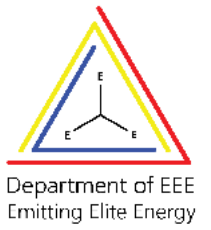
Typical effect:

- Single stage: $f_H = 2 \text{ MHz}$
- Two stages: $f_H = 500 \text{ kHz}$
- Three stages: $f_H = 200 \text{ kHz}$

Thus **gain-bandwidth product** per stage limits final bandwidth.



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Cascade connection

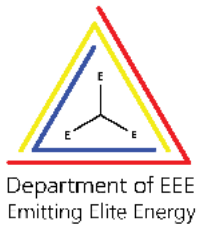
Step 1: Concept

When a **single amplifier stage** doesn't give enough amplification (**gain**), we connect **two or more amplifier stages** in series.

- ☐ This connection is called a **cascade connection** or **multistage amplifier**.



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Step 2: How It Works

- The **output** of one amplifier becomes the **input** of the next.

- Example:**

Stage 1 → Stage 2 → Stage 3

(Output of Stage 1 = Input of Stage 2, and so on.)

Step 3: Voltage Gain Relation

Let each amplifier have a voltage gain:

- A_{V1} for stage 1
- A_{V2} for stage 2
- A_{V3} for stage 3

Then the **total voltage gain** is: $A_{VT} = A_{V1} \times A_{V2} \times A_{V3}$

This means **total gain** = **product of all individual stage gains**.

Step 4: Current Gain

Current gain (A_i) is given by the ratio of output current to input current:

$$A_i = \frac{I_o}{I_i}$$

Using relationships between voltage, current, and resistance:

$$A_i = -A_v \times \frac{Z_i}{R_L}$$

Where:

- A_v = voltage gain
- Z_i = input impedance
- R_L = load resistance

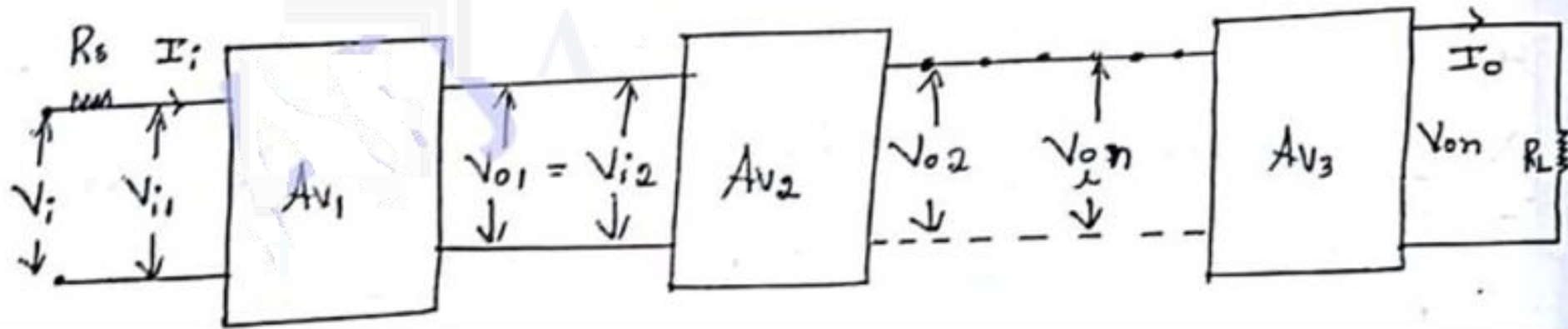
Step 5: Important Notes

- Each stage slightly **loads the next stage**, reducing gain a little (called **loaded gain**).
- The total amplification is **huge** because small gains multiply together.
- Used in **audio amplifiers, signal processing, and communication circuits**.

In Simple Words:

1. **Single amplifier = small output.**
2. Connect many in series → output increases greatly.
3. Multiply individual stage gains → get total gain.
4. Phase reversal occurs (negative sign).
5. Useful for making weak signals strong enough for further use.

Multistage Amplifier with n stages



$$A_v = A_{v1} \times A_{v2} \dots A_{vn}$$

Example:

A given amplifier arrangement has the following voltage gains:

$$A_{V1} = 10, \quad A_{V2} = 20, \quad A_{V3} = 40$$

Find:

- 1 The overall voltage gain, and
- 2 The total voltage gain in dB

Solution:

Step 1:

Overall voltage gain,

$$A_{VT} = A_{V1} \times A_{V2} \times A_{V3}$$

$$A_{VT} = 10 \times 20 \times 40 = 8000$$

Step 2:

Convert each gain into **decibels (dB)** using

$$A_V(dB) = 20 \log_{10}(A_V)$$

$$A_{V1}(dB) = 20 \log_{10} 10 = 20$$

$$A_{V2}(dB) = 20 \log_{10} 20 = 26$$

$$A_{V3}(dB) = 20 \log_{10} 40 = 32$$

Step 3:

Total voltage gain in dB is the sum of all:

$$A_{VT}(dB) = A_{V1}(dB) + A_{V2}(dB) + A_{V3}(dB)$$

$$A_{VT}(dB) = 20 + 26 + 32 = 78 \text{ dB}$$



- Overall Voltage Gain = **8000**
- Total Voltage Gain in dB = **78 dB**

Methods of Cascading Amplifier Stages

In AC coupled amplifiers, stages are coupled together in such a way that DC levels are isolated — only the AC components of a signal are transferred from stage to stage.

In all, there are three methods for coupling of amplifiers:

1. RC Coupling
2. Transformer Coupling
3. Direct Coupling

1.Direct Coupling

In direct coupling, the quiescent DC collector current of the first stage passes to the second stage through its base.

Thus, it is not a true AC coupling method.

- It affects the biasing conditions of the next stage.
- Here, we will discuss the true AC coupled amplifiers and their frequency response.

Example: DC or Low-Frequency Amplifier

- The **output of one transistor** connects **directly** to the **input of the next** (no capacitor or transformer).
 - Suitable for **amplifying slowly varying signals** like **temperature or sensor voltages**.
- ☐ Used in **op-amps** and **instrumentation amplifiers**.

2. RC Coupling

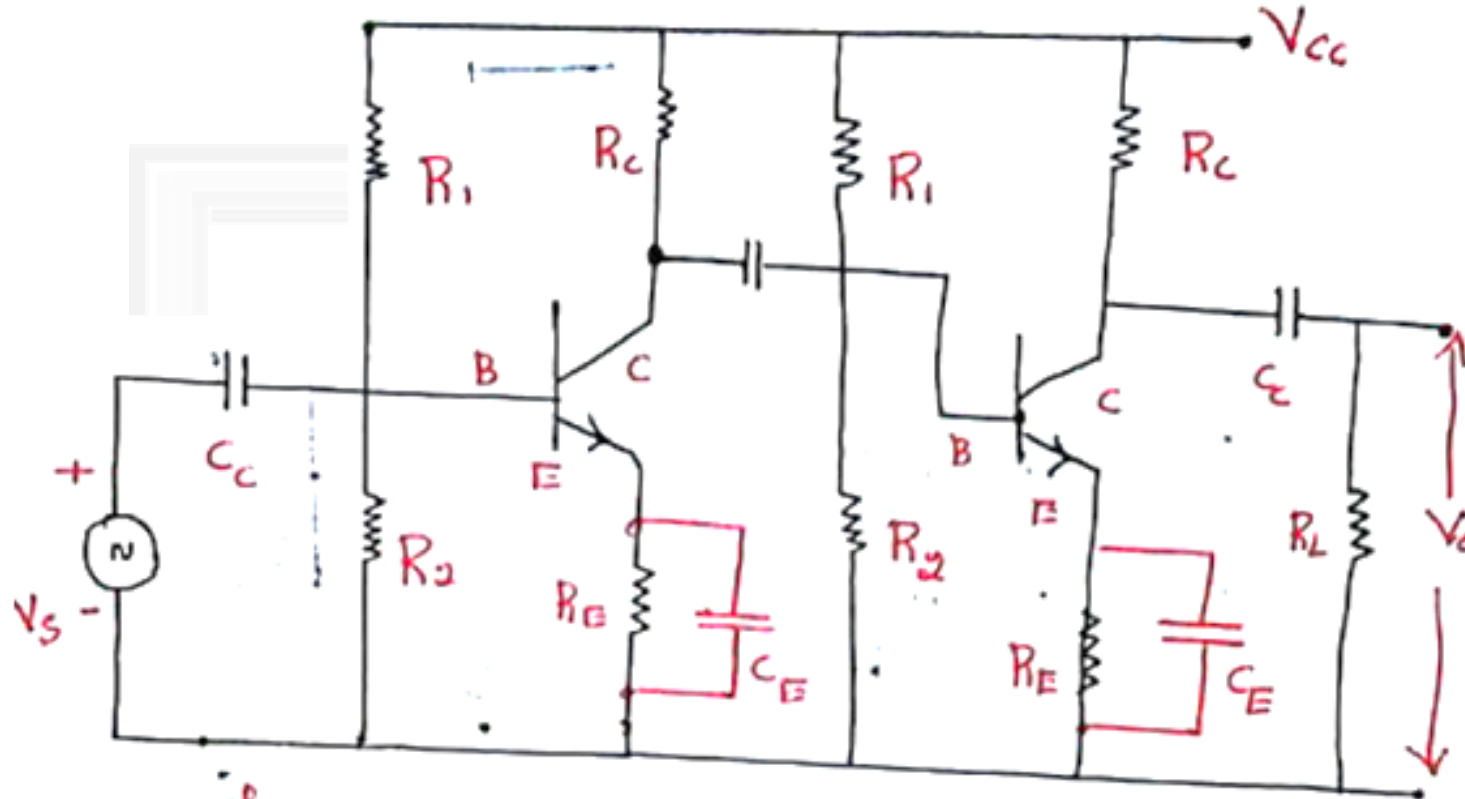


Fig :- Two stage RC coupled Amplifier using transistors

2. RC Coupling

Step-1: Coupling action:

- The AC output of the first stage is passed to the input (base) of the next stage **through a coupling capacitor (C_c)**.
- That coupling capacitor acts as the **load** seen by the output terminal of the first stage for AC signals.

Step-2: DC isolation:

- The coupling capacitor **blocks DC**, so the quiescent (DC) voltage/current of the first stage **does not reach or disturb the bias of the second stage.**
- Therefore the DC operating point (bias) of the next stage is unaffected by the previous stage.

RC Coupling

Step-3: Frequency response (broadband):

- The RC coupling network is **broadband** in nature → it gives a **wide frequency response without sharp peaks**.
- Because of this it is **suitable for audio frequency (AF) amplifiers** covering the whole audio band.

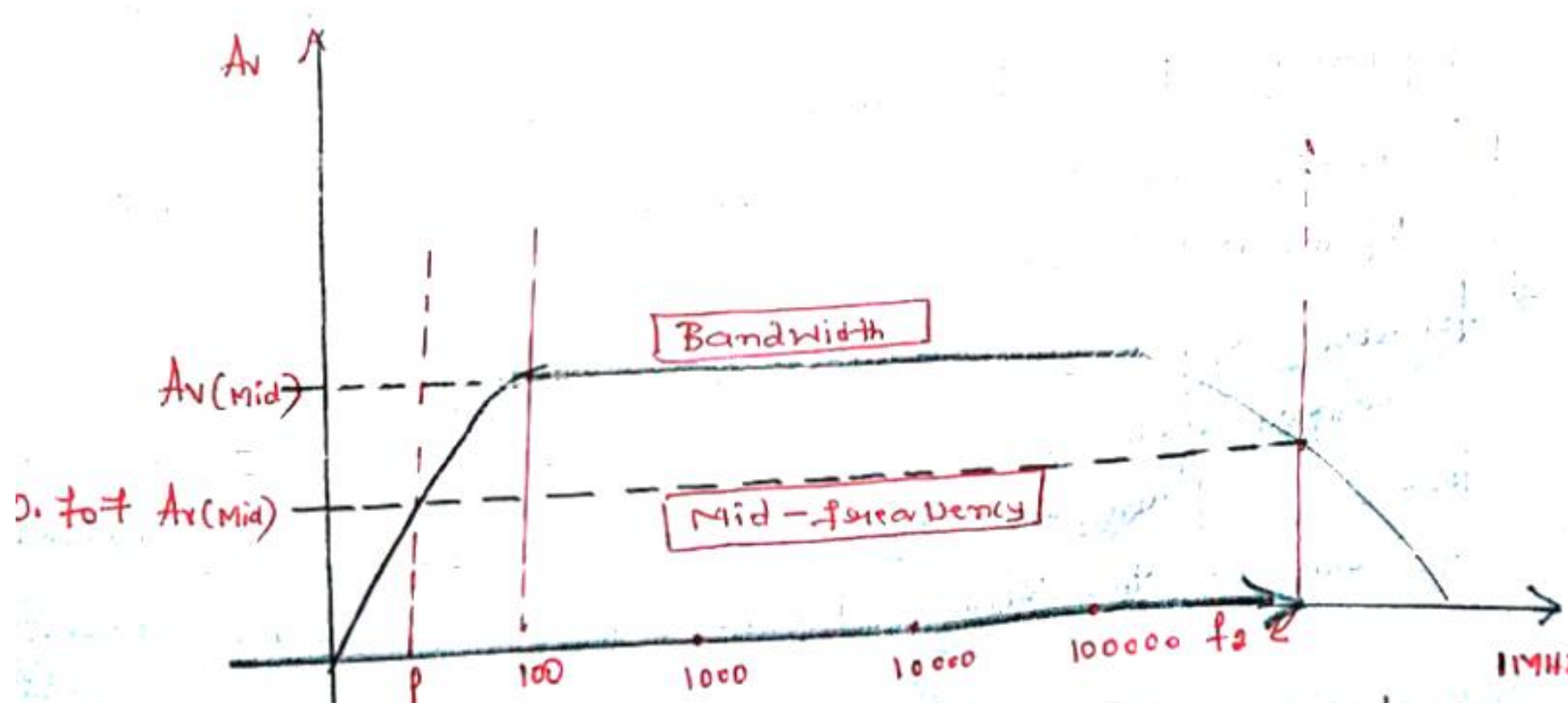
Step-4: Frequency limits (roll-offs):

- Low-frequency cut-off:** response falls at very low frequencies due to the coupling capacitor (too small C_c blocks low frequencies).
- High-frequency cut-off:** response also falls at high frequencies because of shunt/internal capacitances (stray capacitances, transistor capacitances).

Applications:

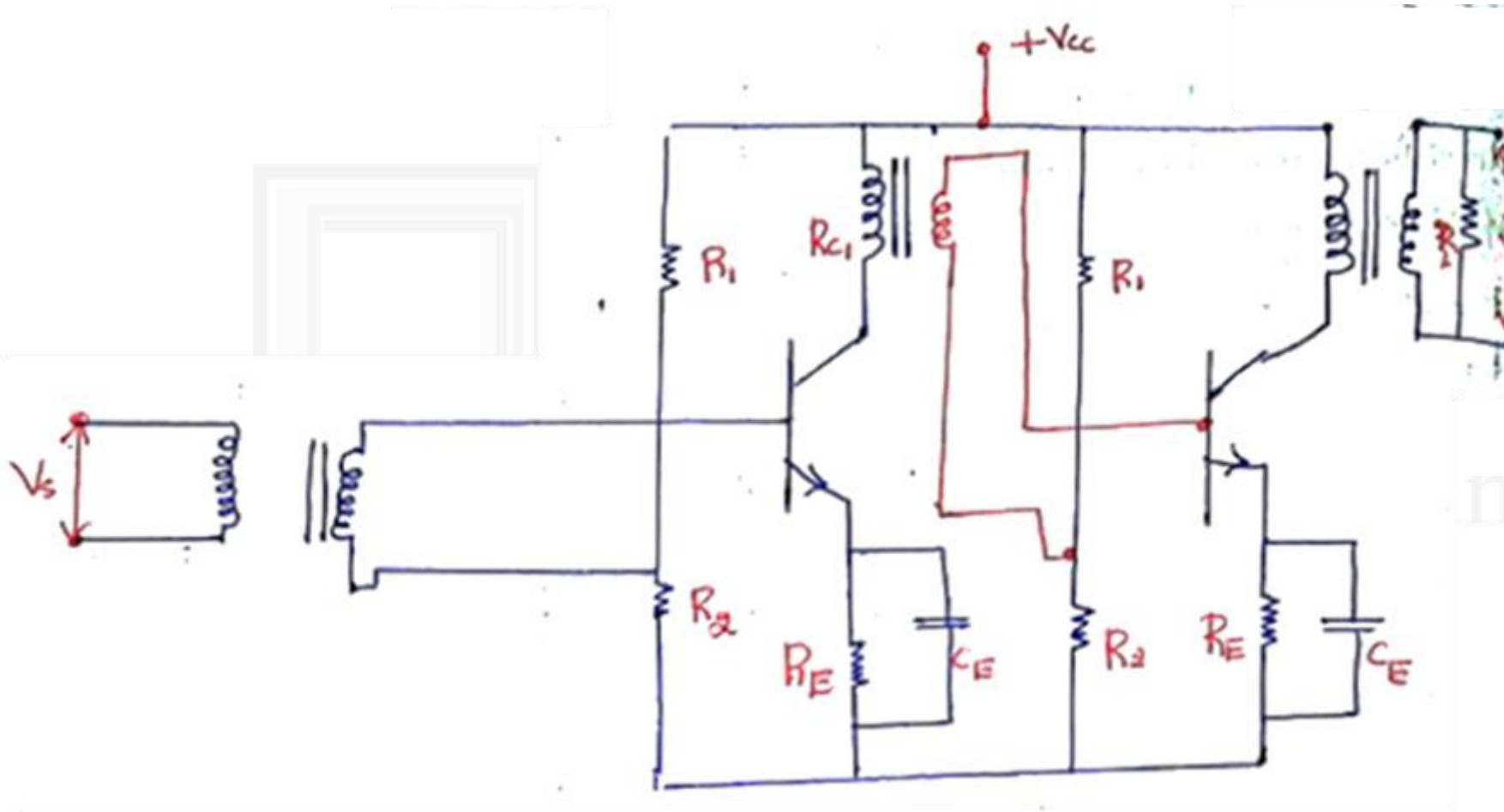
1. Widely used in **small-signal audio amplifiers**.
2. Examples: **tape-recorders, radio receivers, television receivers**, etc.

RC Coupling



Frequency Response

3. Transformer Coupling



1. Coupling Through Transformer

- The output of one amplifier stage goes to the next through a transformer.
- Purpose: **Impedance matching**, especially in audio power amplifiers.

2. DC Isolation

- Transformer blocks DC.
- The next stage's **quiescent point** remains unaffected.

3. Frequency Response Issue

- Leakage inductance and inter-winding capacitance cause **poor bandwidth**.
- Amplifier cannot amplify all frequencies equally well.

4. Resonance at RF Frequencies

- Inter-winding capacitance can cause **natural resonance** at certain frequencies → **very high gain**.
- By adding shunt capacitors, resonance can be tuned to desired **RF frequencies**.

5. Tuned Voltage Amplifier

- When tuned for a specific RF frequency, the circuit becomes a **Tuned Voltage Amplifier**.
- Used in **Radio and TV receivers** to amplify selective RF frequencies.

6. Low DC Resistance Advantage

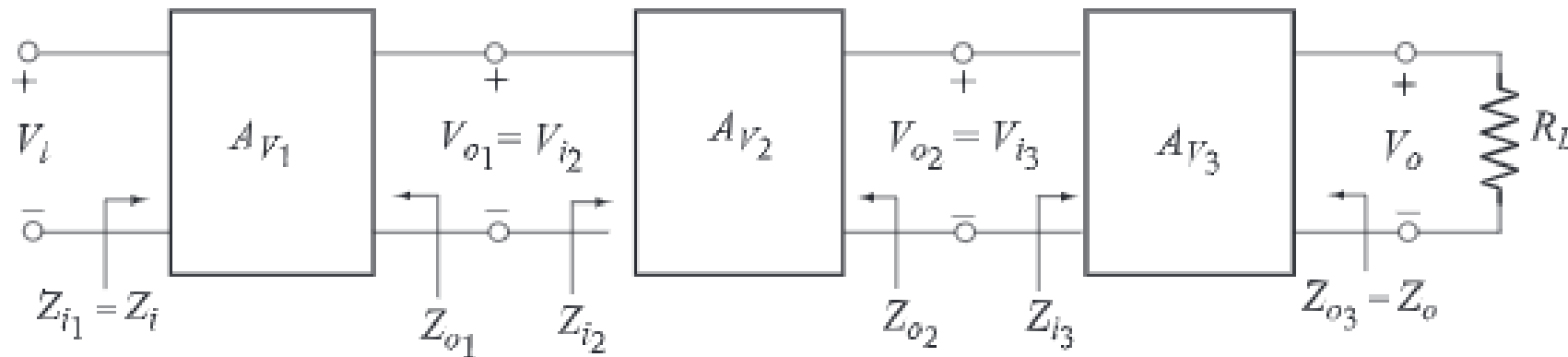
- Transformer winding has very low DC resistance.
- Almost full V_{cc} appears at the collector → higher efficiency.
- No collector resistor → **less power loss**.



Multistage Amplifiers: Cascade Connection

When the amplification from a single stage amplifier is not **sufficient for a particular purpose** or when the input or output impedance is not of suitable magnitude for the intended application, **two or more stages are connected in series.**

The output of one stage is connected as input to the other stage.



Multistage Amplifiers: Cascade Connection

For the cascaded system, the input impedance is that of first stage and output impedance is that of last stage.

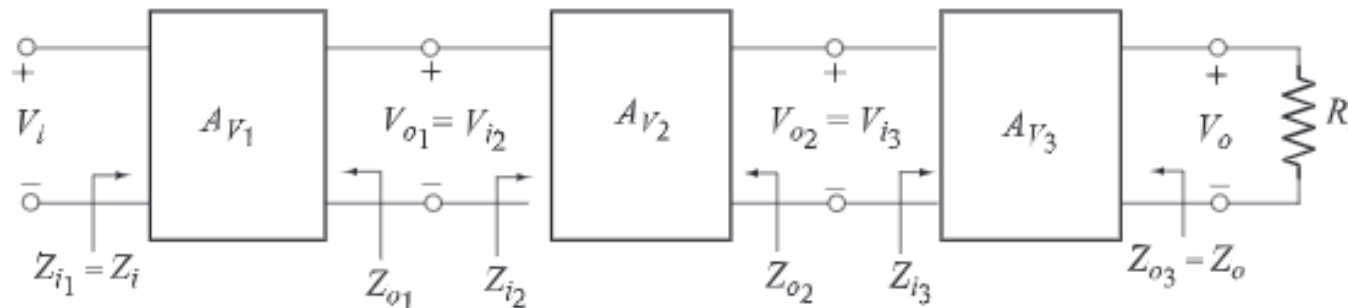
$$Z_i = Z_{i1} \quad \text{and} \quad Z_o = Z_{o3}$$

The output of one stage is connected as input to the other stage.

$$V_{i2} = V_{o1} \quad \text{and} \quad V_{i3} = V_{o2}$$

A total overall voltage gain of the cascading amplifier is:

$$A_{VT} = \frac{V_o}{V_i}$$



Multistage Amplifiers: Cascade Connection

A total overall voltage gain of the cascading amplifier is:

$$A_{VT} = \frac{V_o}{V_I}$$

$$A_{VT} = \frac{V_o}{V_{i3}} * \frac{V_{i3}}{V_{i2}} * \frac{V_{i2}}{V_I}$$

$$V_{i2} = V_{O1} \quad \text{and} \quad V_{i3} = V_{O2}$$

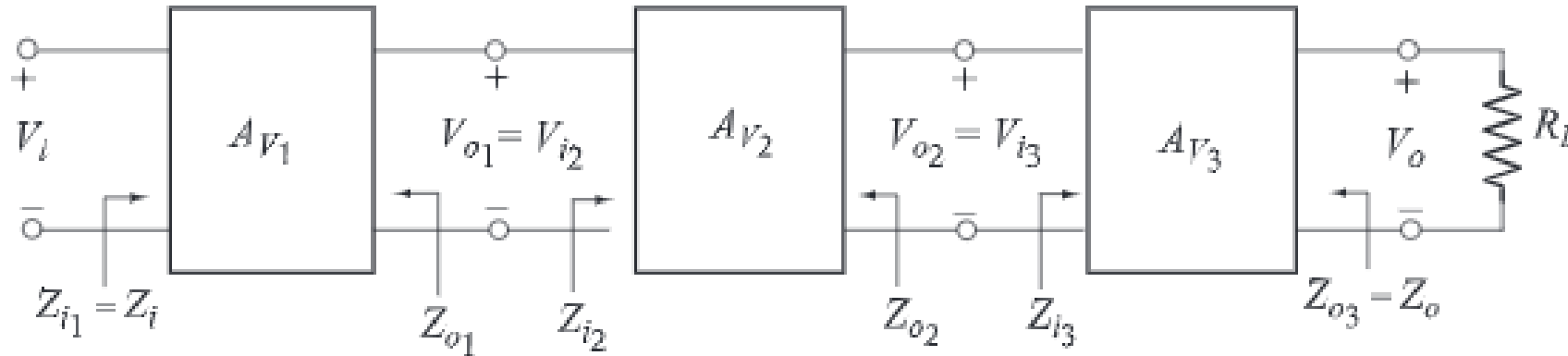
$$A_{VT} = \frac{V_o}{V_{i3}} * \frac{V_{o2}}{V_{i2}} * \frac{V_{o1}}{V_I}$$

$$A_{VT} = A_{V3} * AV_2 * AV_1$$

$$A_{VT} = AV_1 * AV_2 * AV_3$$

For n-cascaded amplifier stages,

$$A_{VT} = AV_1 * AV_2 * AV_3 \text{ ----- } -AV_n$$



A total overall current of the cascading amplifier is:

$$A_{IT} = -AVT * \frac{Z_{i1}}{R_L}$$

the **minus sign** indicates **phase reversal (180° phase shift)** between:

- the **input current**, and
- the **output current** delivered to the load.

Where the phase inversion comes from ??

Most **voltage amplifiers** (common-emitter BJT, common-source MOSFET) introduce **one phase inversion**:

$$V_o = -A_v V_i$$

If the cascaded system contains **one inverting stage**, the overall transfer becomes **negative**.

Even if you cascade many stages, the **total number of inversions** determines the sign:

- **Odd number of inverting stages → negative sign**
- **Even number of inverting stages → positive sign**



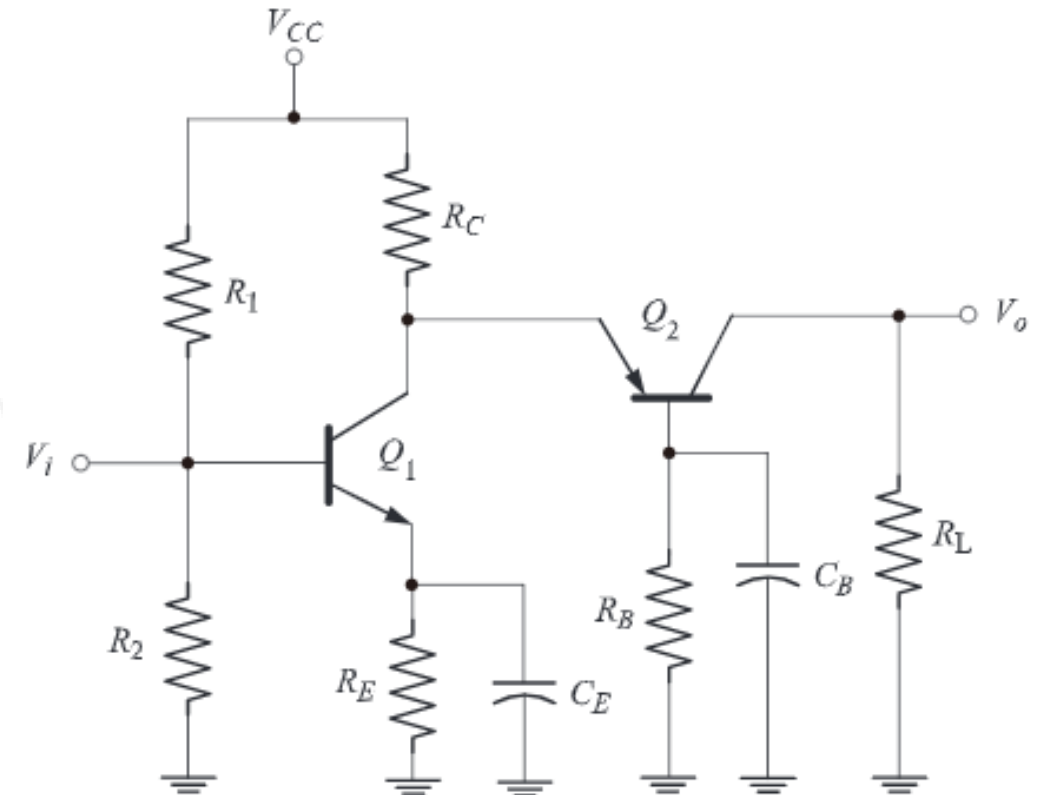
Multistage Amplifiers: Cascade Connection

In this connection the output of **Common Emitter (CE) stage drives the input of Common Base (CB) stage.**

1. Low input capacitance
2. High input impedance (provided by CE)
3. High output impedance (provided by CB)
4. Excellent high frequency response

The voltage gain of CE stage is very low.

R_B is used to limit the bias current of Q_2 .



Solved Examples

1. For the cascaded arrangement shown below, determine

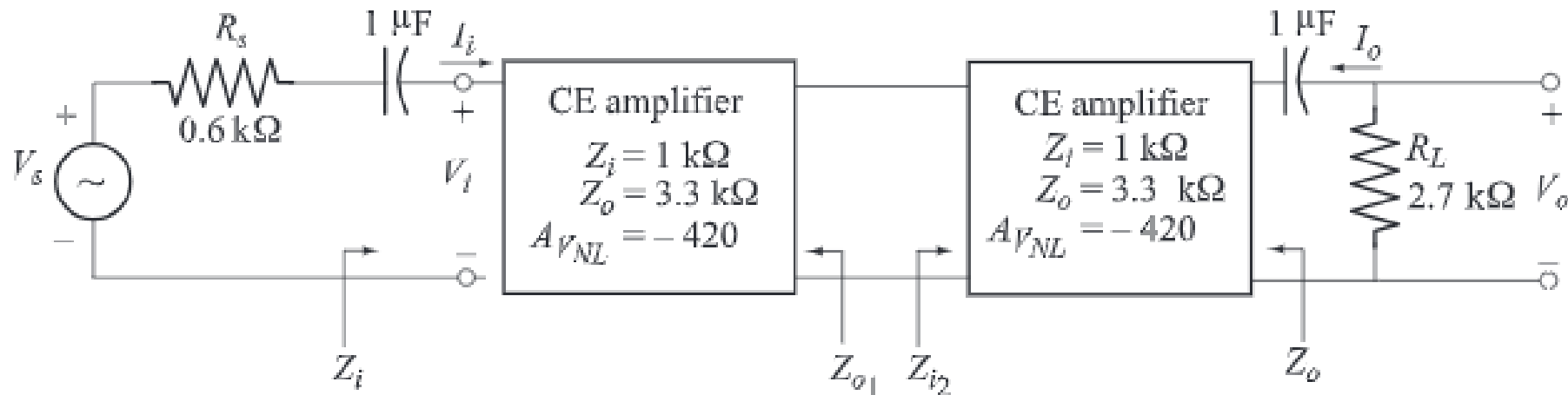
- The loaded voltage gain of each stage
- The total gain of the system, A_V and A_{VS}
- The current gain of each stage
- The total current gain

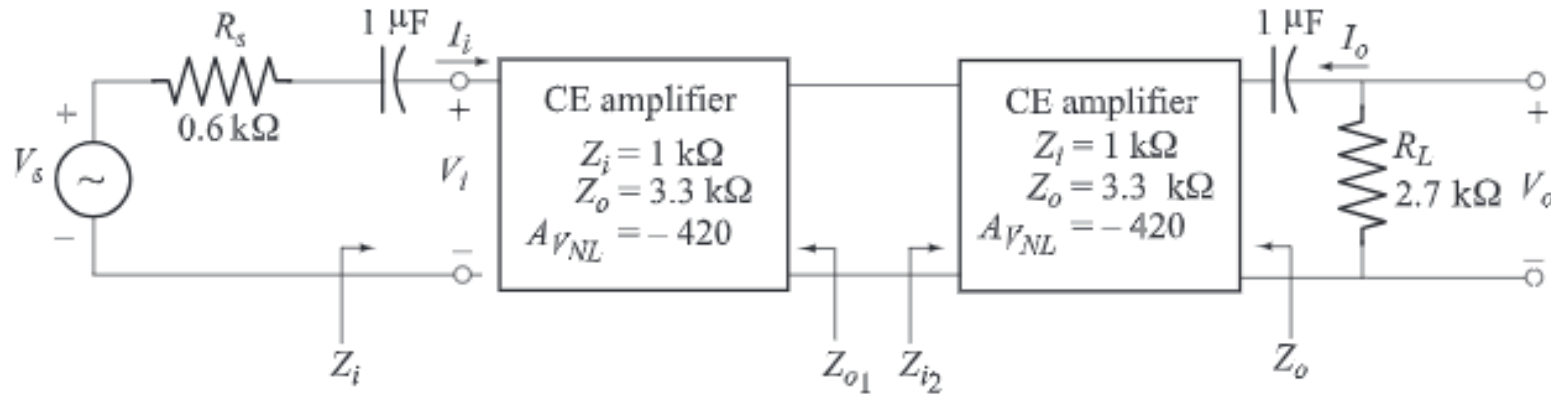
Given:

Stage 1: $Z_i = 1\text{k}\Omega$, $Z_o = 3.3\text{k}\Omega$, $A_{VNL} = -420$

Stage 2: $Z_i = 1\text{k}\Omega$, $Z_o = 3.3\text{k}\Omega$, $A_{VNL} = -420$

$R_S = 0.6\text{k}\Omega$, $R_L = 2.7\text{k}\Omega$





a. The loaded gain of each stage

For the emitter follower stage the load is Z_{i2} .

$$A_{V1} = \frac{V_{O1}}{V_{i1}} = \frac{Z_{i2}}{Z_{i2} + Z_{O1}} A_{VNL}$$

$$A_{V1} = \frac{1k}{1k + 3.3k} * -420 = -97.67$$

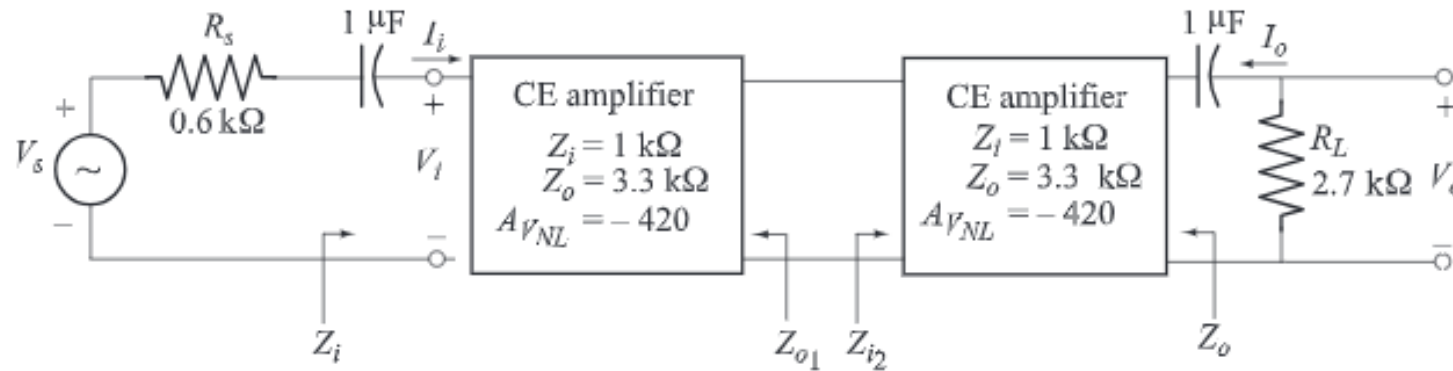
$$A_{V2} = \frac{V_{O2}}{V_{i2}} = \frac{R_L}{R_L + Z_{O2}} A_{VNL}$$

$$A_{V2} = \frac{2.7k}{2.7k + 3.3k} * -420 = -189$$

b. Total voltage gain

$$A_{VT} = A_{V1} * A_{V2}$$

$$A_{VT} = -97.67 * -189 = 18.45 * 103$$



$$A_{VS} = \frac{Z_{i1}}{Z_{i1} + R_S} A_{VT}$$

$$A_{VS} = \frac{1k}{1k + 0.6k} * 18.45 * 10^3 = 11.53 * 10^3$$

c. Current gain of each stage

$$A_{I1} = -A_{V1} * \frac{Z_{i1}}{Z_{i2}}$$

$$A_{I1} = -(-97.67) * \frac{1k}{1k} = 97.67$$

$$A_{I2} = -A_{V2} * \frac{Z_{i2}}{R_L} = -(-189) * \frac{1k}{2.7k} = 70$$

$$A_{IT} = A_{I1} * A_{I2} = 97.67 * 70 = 6.84 * 10^3$$

Analysis for CE-CE mode

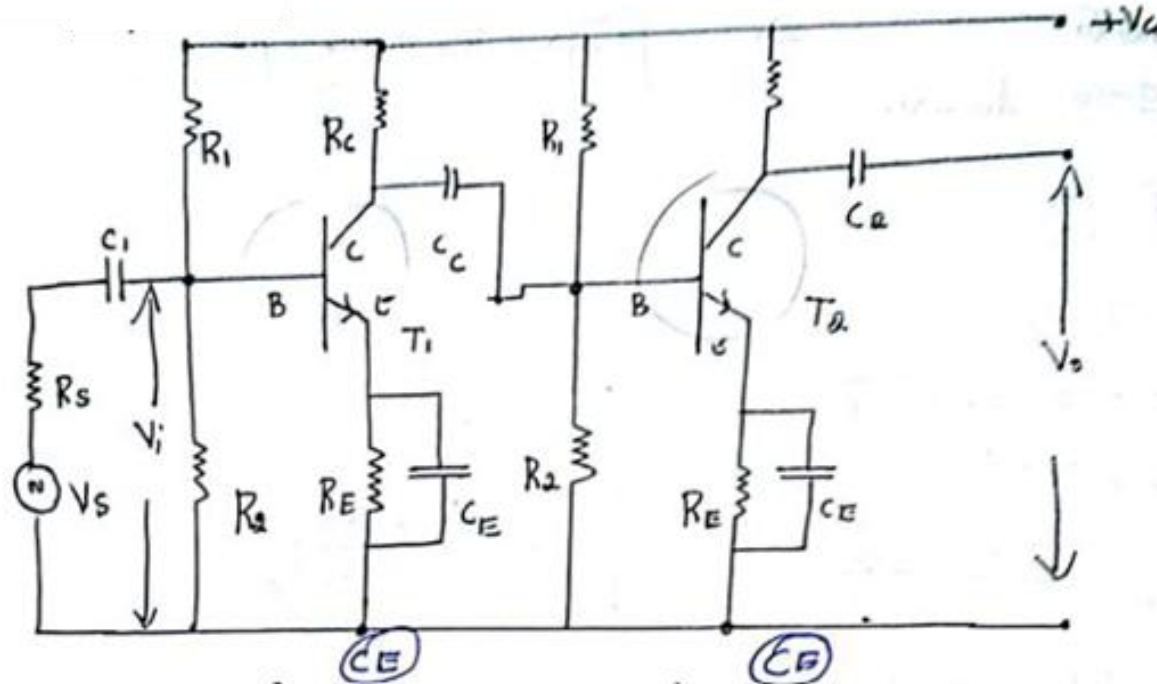


Fig shows two stage RC coupled CE-CE cascaded.



Amplifier with their biasing arrangement. The coupling capacitors provide DC isolation b/w 2 stages of amp as a **short ckt equivalent for AC response**.

1. V_B

Voltage divider gives the base voltage.

2. V_{BE}

Used to find base-emitter drop.

3. I_E

Emitter current = emitter voltage / emitter resistor.

4. Z_{i1}

Input impedance comes from parallel combination of resistors and transistor input.

5. & 6. Voltage Gain of Each Stage

CE amplifier gain = $-(\text{collector resistance} / \text{internal emitter resistance})$.

Second stage similar.

7. Total Gain A_{VT}

Cascade $\rightarrow$ multiply gains of both stages.



8. Output Voltage

Output = total gain $\times$ input.

9. Output Impedance

CE output impedance equals R_C .

10. Loaded Gain A_V

Load reduces the total gain because it forms a divider with amplifier output impedance.

Important Equations (CE)

1. $V_B = \frac{R_2}{R_1 + R_2} \times V_{CC}$
2. $V_E = V_B - V_{BE}$
3. $I_E = \frac{V_E}{R_E}$

To determine A_v , Z_o , Z_i :

4. $Z_{i1} = Z'_{i2} = R_1 \parallel R_2 \parallel \beta r_e$
5. $A_{v1} = -\frac{R_{C1} \parallel Z'_{i2}}{r_e}$
6. $A_{v2} = -\frac{R_{C2}}{r_e}$

Cascade (CE–CE) Amplifier

7. $A_{VT} = A_{v1} \times A_{v2}$
8. $V_o = A_{VT} \times V_i$
9. $Z_{o2} = R_C$
10. $A_V = A_{VT} \times \frac{R_L}{R_L + Z_{o2}}$

Solved Examples

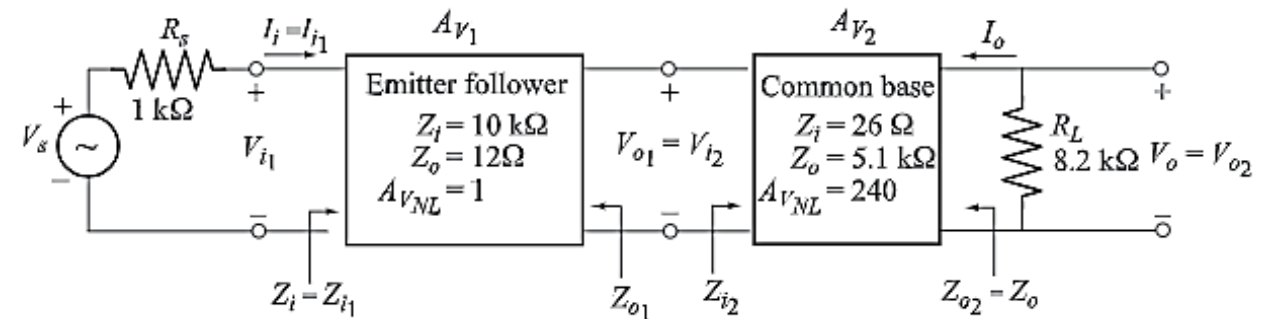
1. For the cascaded arrangement shown below, determine
 - a. The loaded voltage gain of each stage
 - b. The total gain of the system, A_v and A_{v_s}
 - c. The total current gain of the system

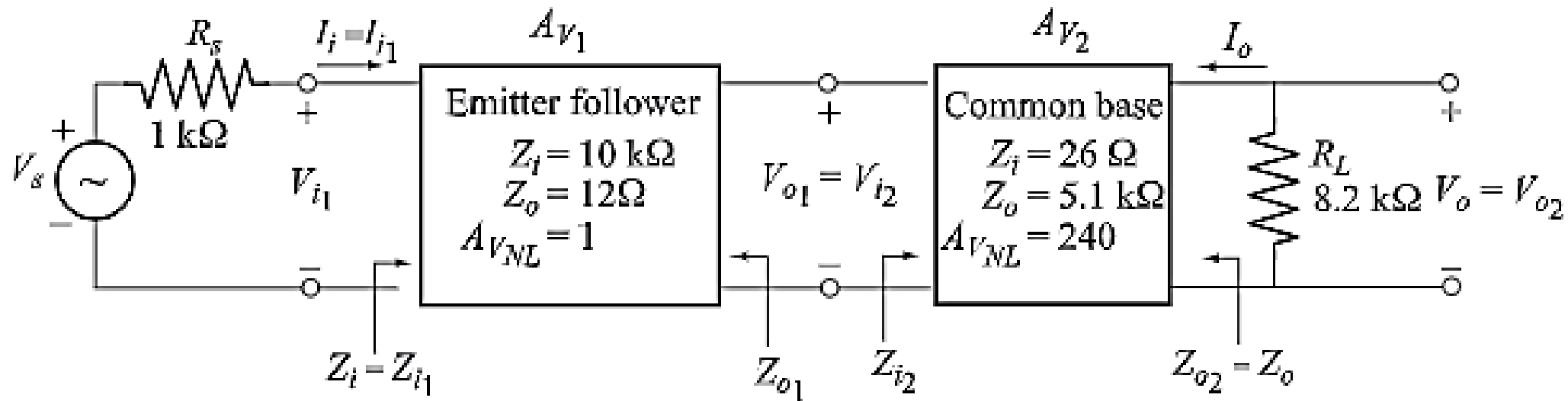
Given:

Stage 1: $Z_i = 10\text{k}\Omega$, $Z_o = 12\Omega$, $A_{v_{NL}} = 1$

Stage 2: $Z_i = 26\Omega$, $Z_o = 5.1\text{k}\Omega$, $A_{v_{NL}} = 240$

$R_s = 1\text{k}\Omega$, $R_L = 8.2\text{k}\Omega$





a. The loaded gain of each stage

For the emitter follower stage the load is Z_{i2} .

$$A_{V1} = \frac{V_{O1}}{V_{i1}} = \frac{Z_{i2}}{Z_{i2} + Z_{O1}} A_{VNL}$$

$$A_{V1} = \frac{26}{26 + 12} * 1 = \mathbf{0.684}$$

$$A_{V2} = \frac{V_{O2}}{V_{i2}} = \frac{R_L}{R_L + Z_{O2}} A_{VNL}$$

$$A_{V2} = \frac{8.2k}{8.2k + 5.1k} * 240 = \mathbf{147.97}$$

b. Total voltage gain

$$A_{VT} = A_{V1} * A_{V2}$$

$$A_{VT} = 0.684 * 147.97 = \mathbf{101.20}$$

$$A_{VS} = \frac{Z_{i1}}{Z_{i1} + R_S} A_{VT}$$

$$A_{VS} = \frac{10k}{10k + 1k} * 101.20 = \mathbf{92}$$

c. Total current gain

$$A_{IT} = -A_{VT} * \frac{Z_{i1}}{R_L}$$

$$A_{IT} = -(101.20) * \frac{10k}{8.2k} = \mathbf{-123.41}$$

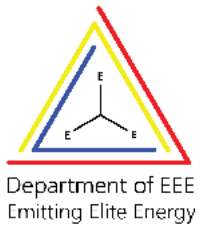
A **Darlington pair** is a combination of **two transistors** connected together so that the current amplified by the first transistor is further amplified by the second.

Why it is used:

1. **Very high current gain (β)** – Multiplication of both transistor gains.
2. **Used to drive high-current loads** from a small input signal.
3. **High input impedance**, so it does not load the previous stage.



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Simple Example:

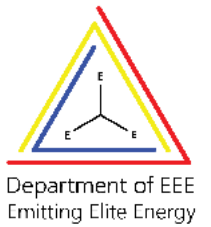
If T1 has gain 50 and T2 has gain 100,

Total gain $\approx 50 \times 100 = 5000$ $\rightarrow$ useful for sensors, switching relays, motors, LEDs, etc.

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DIY Amplifier Darlington 2N3055 Transistor Extremely powerful Using Output Capacitors

<https://www.youtube.com/watch?v=a0cZWrm4-y0>

Darlington Amplifier

β_1 = Current gain of Q_1

β_2 = Current gain of Q_2

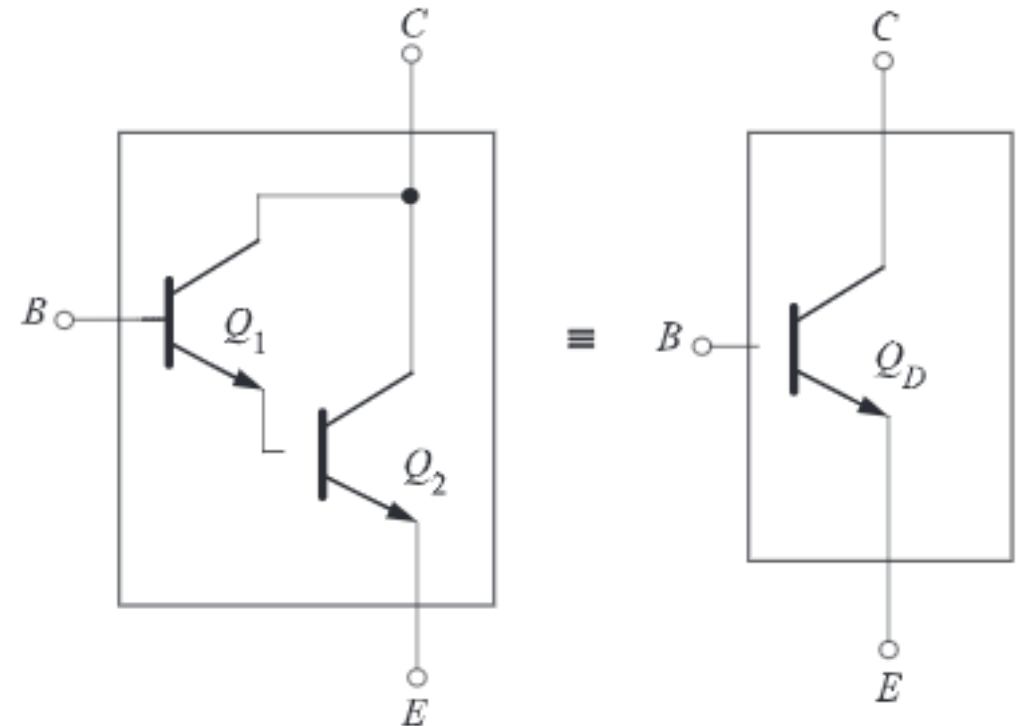
The current gain of Darlington Transistor:

$$\beta_D = \beta_1 * \beta_2$$

If $\beta_1 = \beta_2 = \beta$

$$\beta_D = \beta^2$$

A Darlington connection acts as single transistor with a **large current gain**.



Darlington Amplifier: DC Bias

Step-1: Applying KVL to the base circuit we get;

$$V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$$

$$V_{BE} = V_{BE1} + V_{BE2}$$

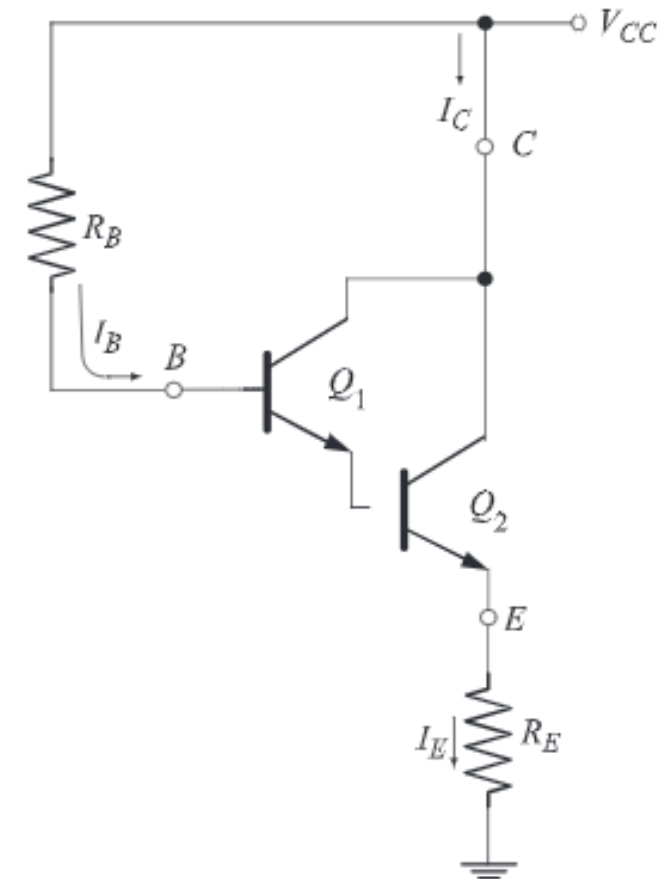
Step-2: $I_E = (1 + \beta_D) I_B \approx \beta_D I_B$

$$I_B R_B = V_{CC} - V_{BE} - I_E R_E$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (1 + \beta_D) R_E}$$

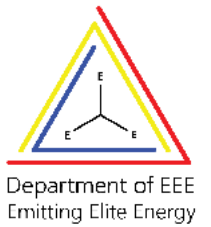
Step-3: $V_E = I_E * R_E$

$$V_B = V_{BE} + V_E$$



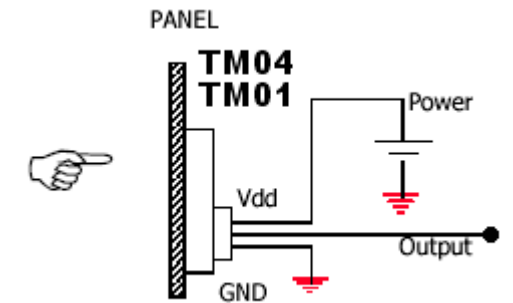


A T M E
College of Engineering



Darlington equations tell you how the circuit acts as a **high-current**, **low-output-impedance** **voltage follower** with **huge current gain** and **very high input impedance**.





1. Touch Sensor Lamps / Touch Switches

When you touch a metal plate, the current from your finger is extremely small (microamps).

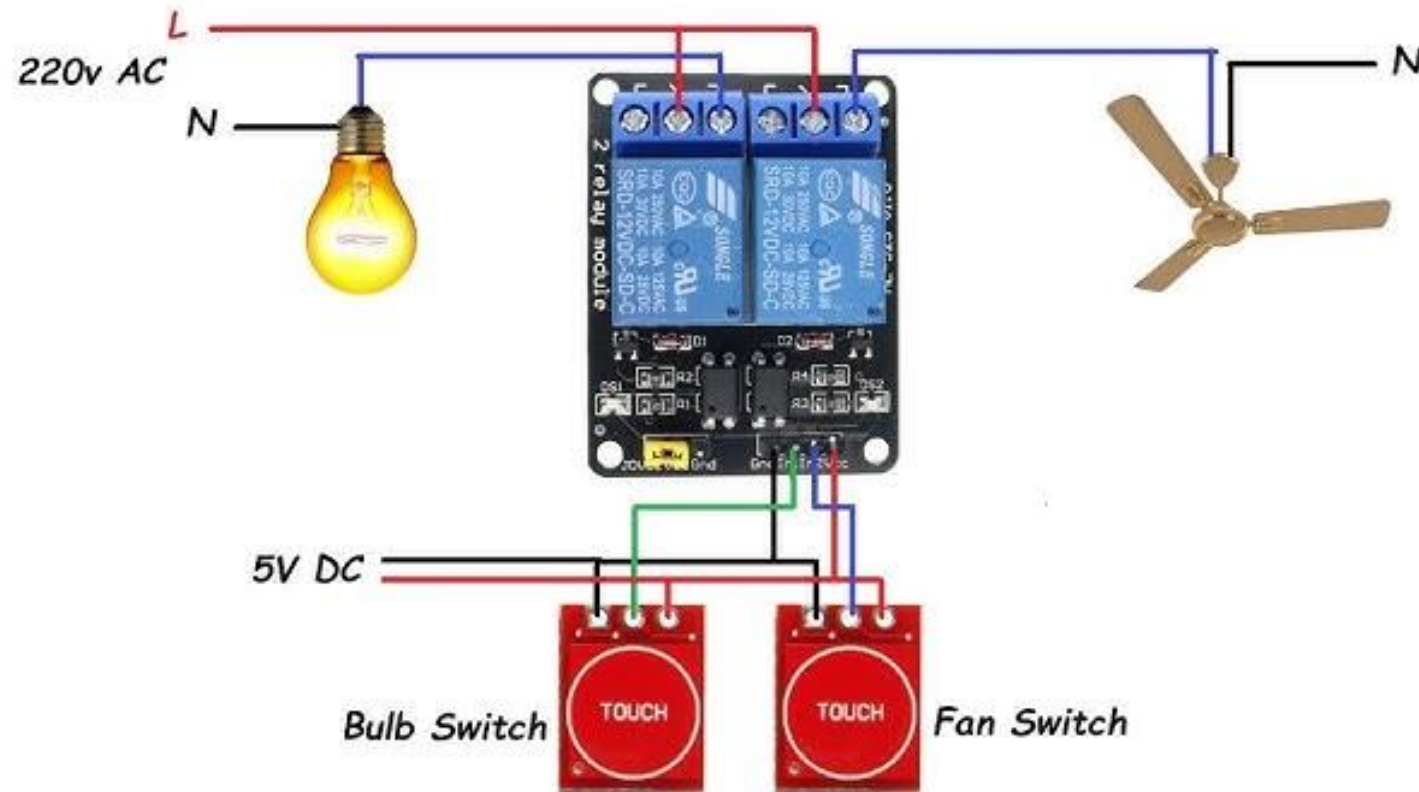
A Darlington pair amplifies this tiny current enough to:

- ✓ turn ON/OFF a lamp
- ✓ trigger a relay

Why Darlington?

Because it has **very high current gain** and works with weak signals.

How to make Touch Switch

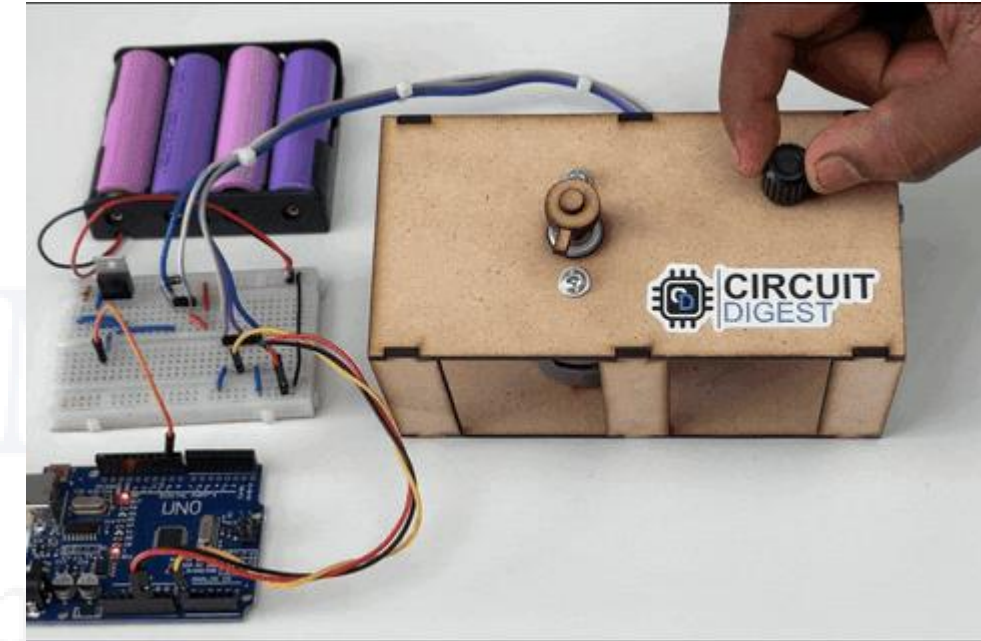


2. Microcontroller → Motor Drive (Without Driver IC)

Arduino/ESP pins give only **20–40 mA**, which is too small for:

- DC motors
- Fan motors
- Vibrator motors

A Darlington pair allows the microcontroller to drive motors safely.



Why Darlington?

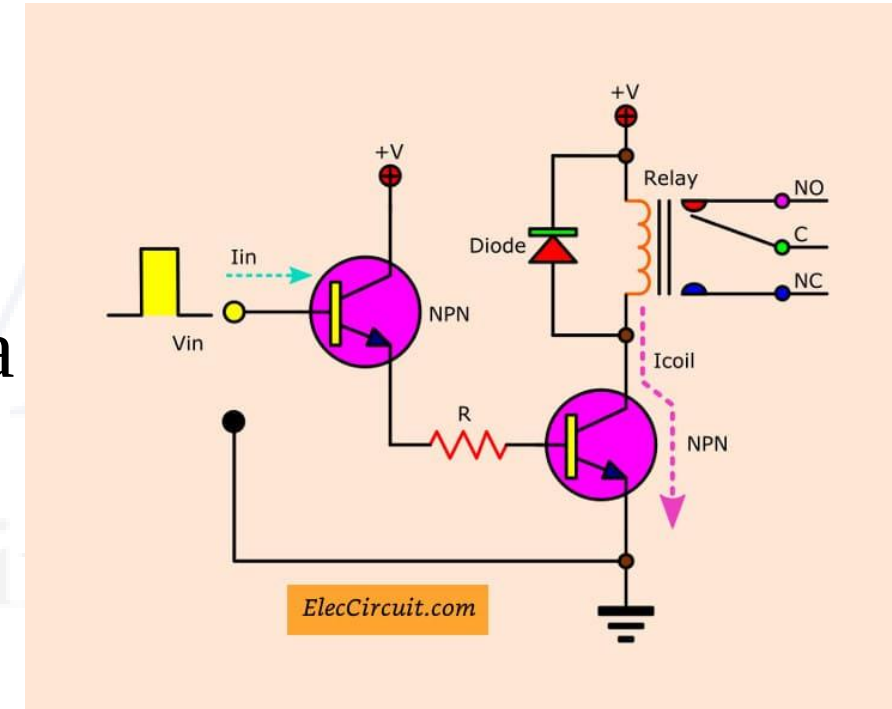
It converts **small input current** → **big output current**.

3. Relay Drivers

Relays need **50–200 mA**, but sensors/microcontrollers cannot provide this. Using a Darlington, you can directly energize a relay coil.

Where used in reality?

- Home automation relay boards
- Power switching modules
- Security systems



4. Audio Amplifier Input Stages

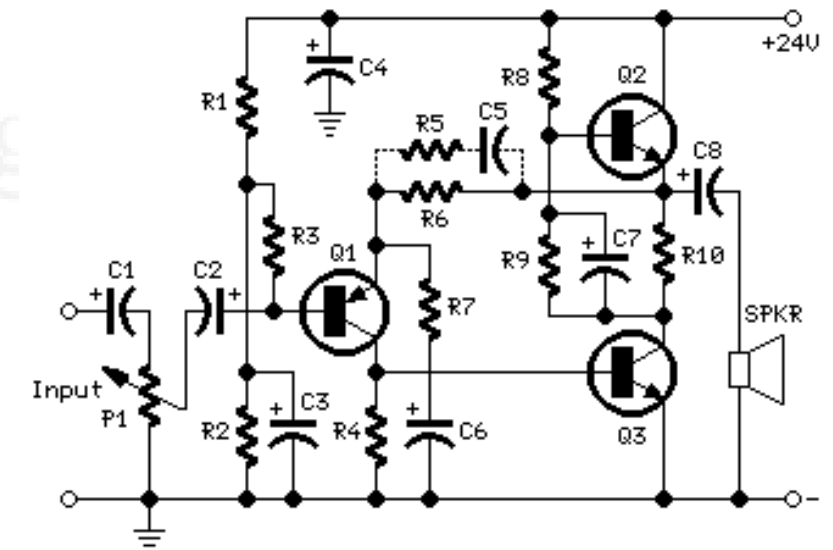
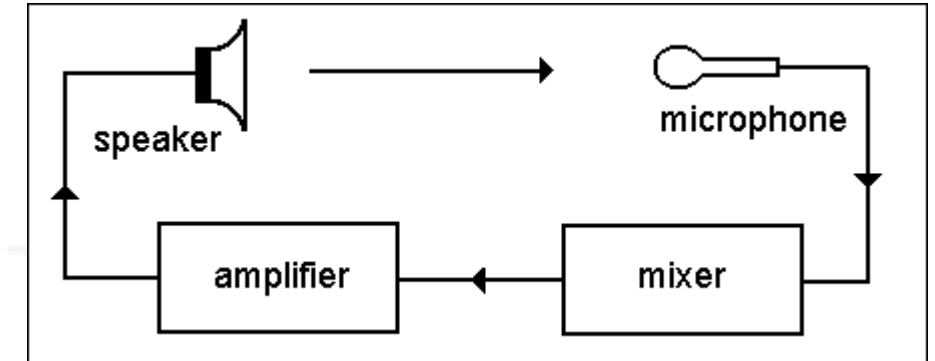
The first stage of many audio amplifiers uses a Darlington pair.

Why?

- ✓ High input impedance → does not load the audio source
- ✓ Good current boost for next amplifier stage

Used In:

- Speaker amplifiers
- Headphone amps
- Radio receivers



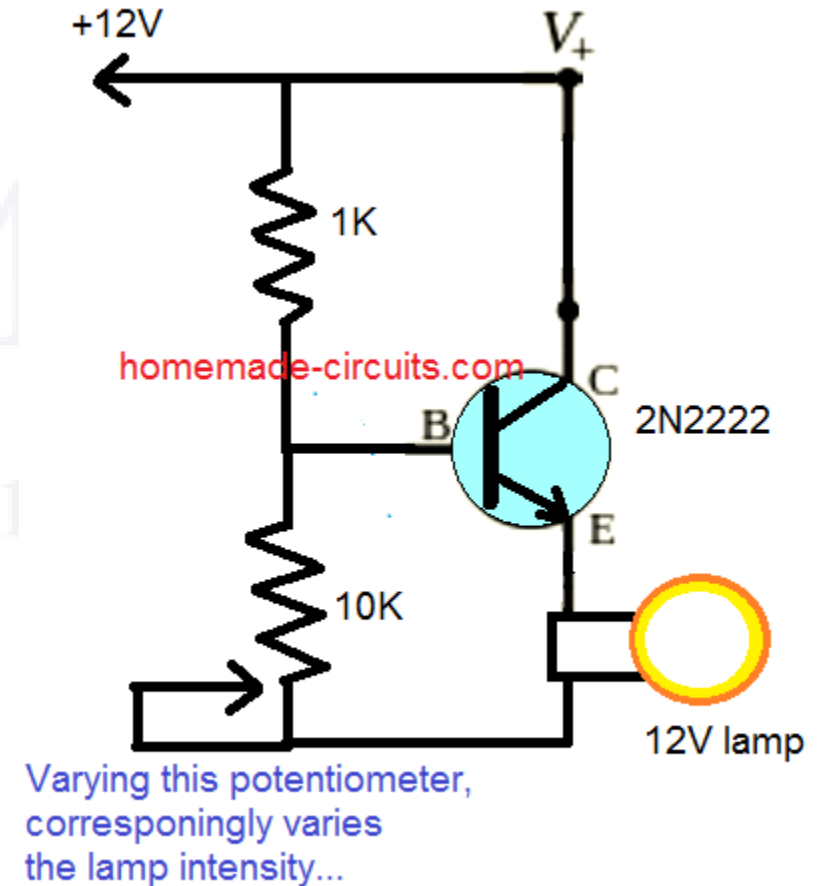


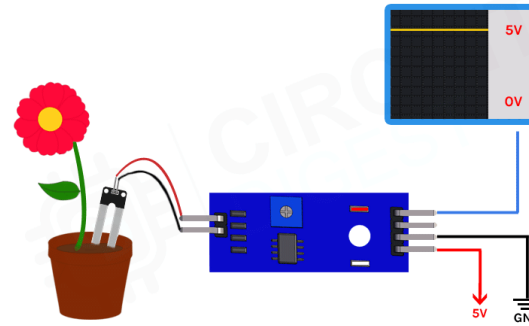
5. Light Dimmer, Fan Regulator (Old Designs)

Darlington pairs were used to control the phase/power of AC loads before **MOSFETs** became popular.

Used In:

- Old fan speed controllers
- Old light dimmers





6. Liquid Level / Soil Moisture Sensors

Sensor current is very weak.

Darlington pair boosts it to drive:

- buzzer
- pump
- LED indicator
- controller input



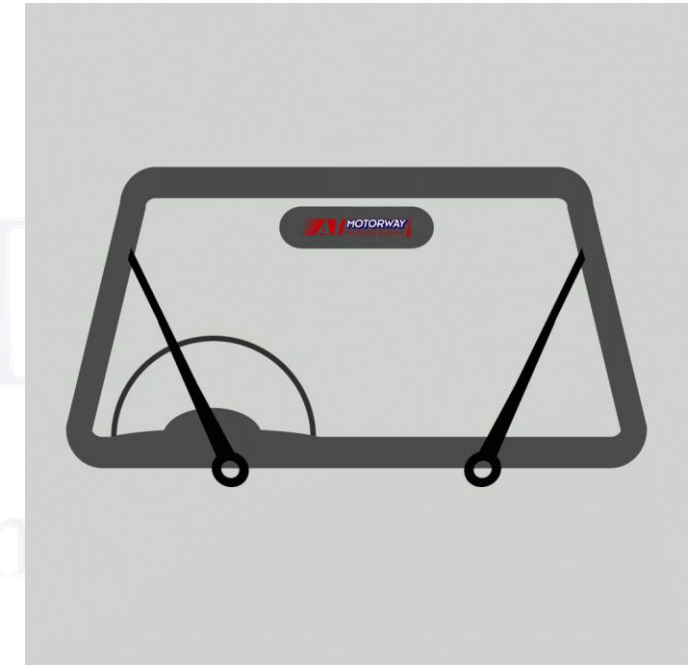
7. Automotive Electronics

Car sensors generate small currents.

Darlington pairs amplify signals for:

- fuel injectors
- ignition control
- cooling fan control
- wiper motor control

Must handle large currents → Darlington pair is ideal.



8. Temperature-Controlled Fans

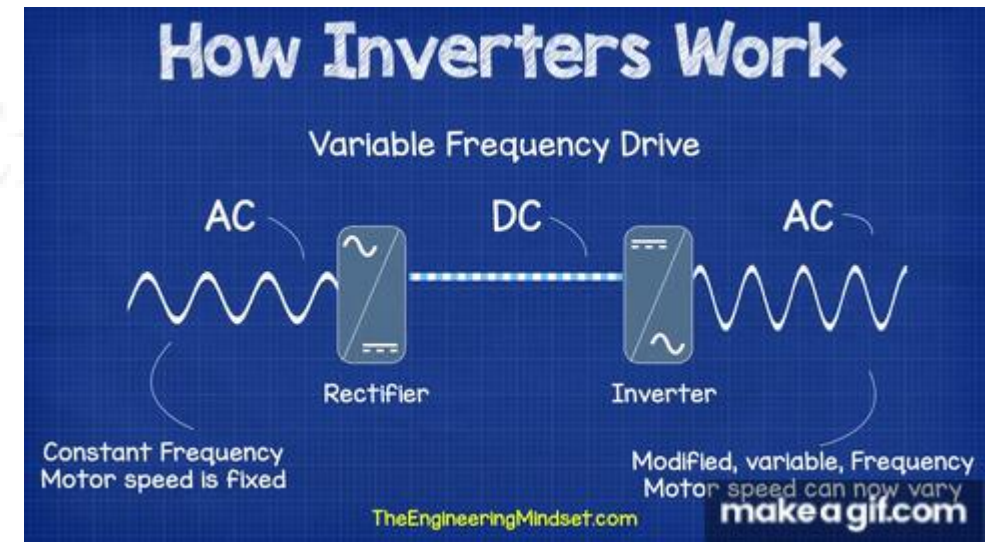
Temperature sensors like NTC produce low current.

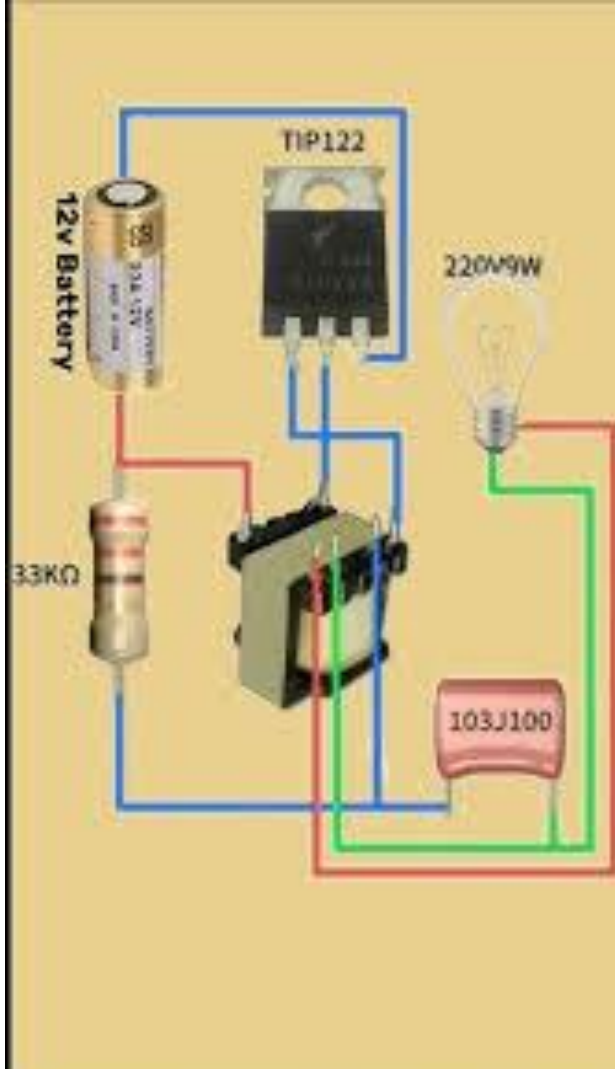
Darlington amplifies it to drive:

- 12V fans
- blower motors

Used in:

- CPU coolers
- Power supply cooling
- Inverters





TIP122 Darlington power transistor to convert **12V DC to approximately 220V AC** for powering a **9W AC bulb**.

The setup includes a 12V battery as the power source, a 33kΩ resistor for base biasing, and a 103J100 capacitor (0.01μF, 100V) for oscillation and filtering. The transistor drives a center-tapped transformer, with one end connected to the collector and the center tap connected to the positive terminal of the battery. The switching action of the TIP122, controlled by the RC network, induces an alternating voltage in the secondary winding of the transformer, thus generating a stepped-up AC voltage suitable for low-power AC loads.

9. Voltage Regulators (LM317 + Darlington)

To increase current output of LM317 from 1.5A $\rightarrow$ up to 10A, a Darlington transistor is added.

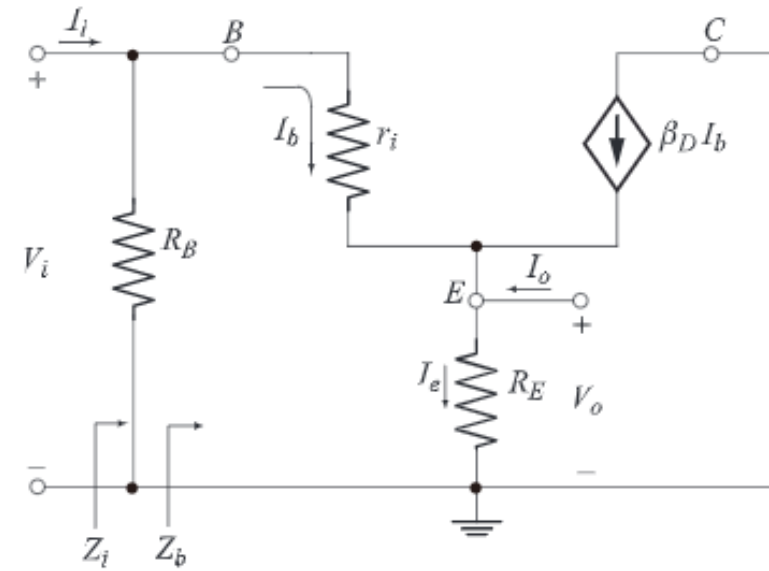
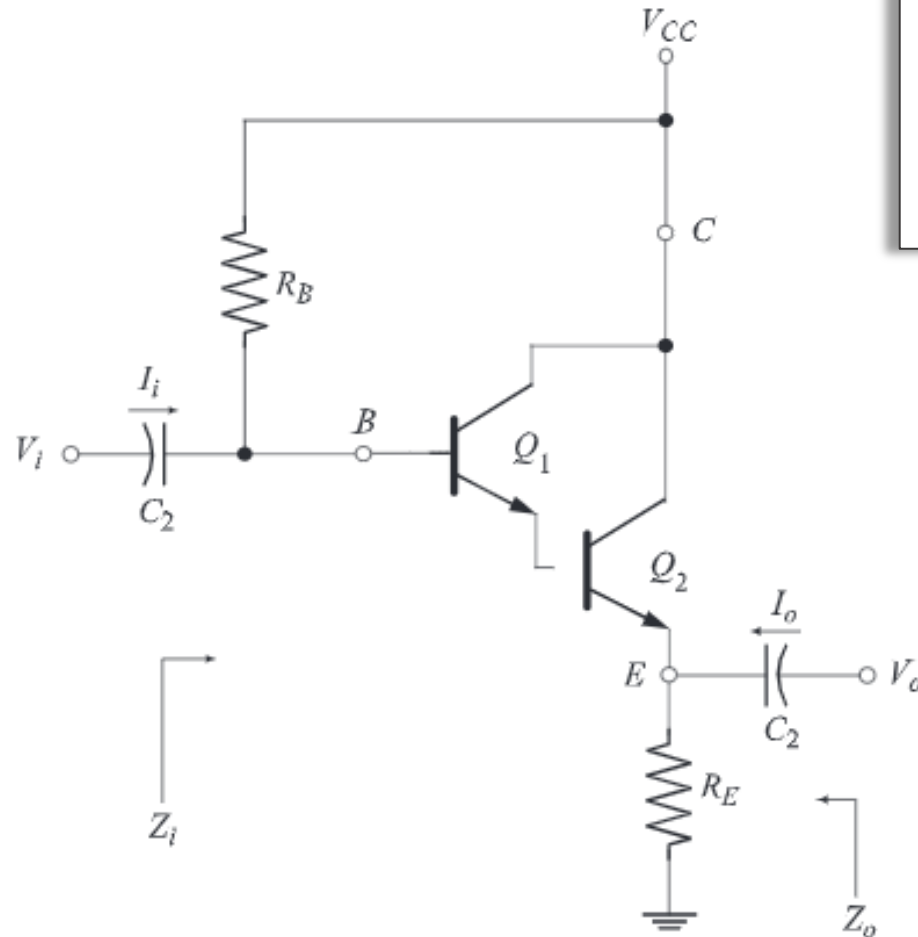
Used in:

- Battery chargers
- Adjustable power supplies

Darlington Emitter Follower

The Darlington transistor is replaced by

1. Input resistance r_i between base and emitter.
2. Controlled current source between collector and emitter terminals.



AC equivalent circuit

1. AC Voltage Gain (A_v)

$$A_v = V_o / V_i$$

Voltage gain ≈ 1 because emitter followers nearly copy input voltage to the output.

1. AC Voltage Gain (A_V)

We start from the small-signal model.

Step 1 – Output voltage

Output voltage appears across R_E :

$$V_O = I_e R_E$$

Step 2 – Express I_e in terms of I_b

For a Darlington pair:

$$I_e = (1 + \beta_D) I_b$$

Substitute:

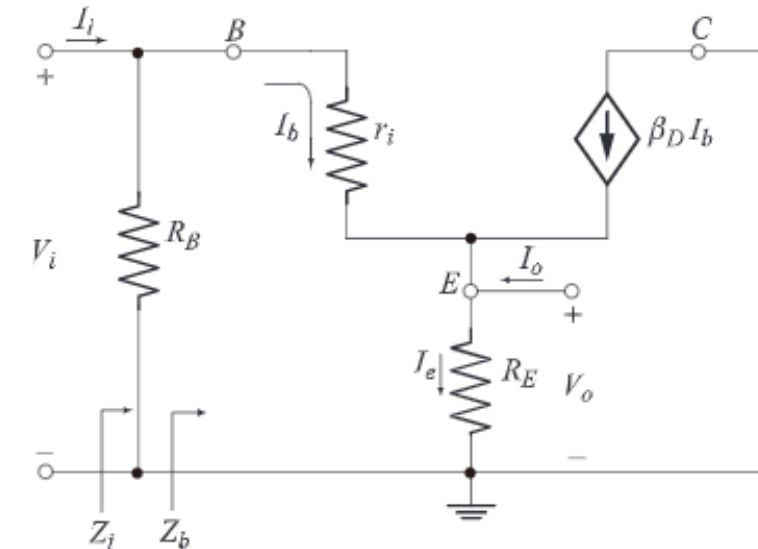
$$V_O = (1 + \beta_D) I_b R_E$$

Step 3 – Input Voltage

Apply KVL at input loop:

$$V_I = I_b r_i + I_e R_E$$

Substitute $I_e = (1 + \beta_D) I_b$



Substitute $I_e = (1 + \beta_D)I_b$

$$V_I = I_b r_i + (1 + \beta_D)I_b R_E$$

Factor out I_b :

$$V_I = I_b [r_i + (1 + \beta_D)R_E]$$

Step 4 – Voltage Gain

$$A_V = \frac{V_O}{V_I} \quad \frac{V_O = (1 + \beta_D)I_b R_E}{V_I = I_b [r_i + (1 + \beta_D)R_E]}$$

$$A_V = \frac{(1 + \beta_D)R_E}{\cancel{r_i} + (1 + \beta_D)R_E}$$

Step 5 – Approximation

$$\text{Since } (1 + \beta_D)R_E \gg r_i \quad \longrightarrow \quad A_V \approx 1$$

BRIEF EXAMPLES

1. Voltage Gain Example

If $(1 + \beta_D)R_E = 10 \text{ k}\Omega$ and $r_i = 50 \Omega$:

$$A_V \approx \frac{10k}{50 + 10k}$$

$$\approx 0.995 \approx 1$$

Interpretation: Output voltage $\approx$ input voltage
→ voltage buffer.

2. AC Input Impedance (Z_I)

$$Z_I = ?$$

Step 1 – Using V_I Expression

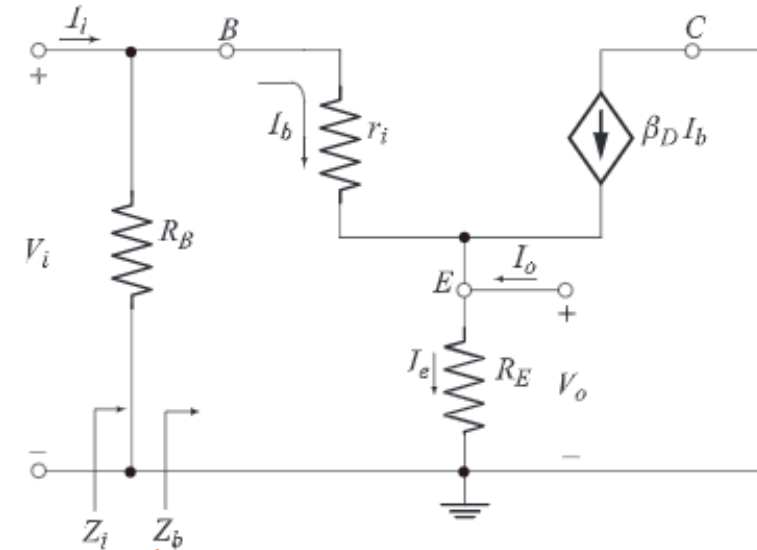
From above:

$$I_e = (1 + \beta_D)I_b$$

$$V_I = I_b r_i + I_e R_E$$

$$V_I = I_b [r_i + (1 + \beta_D)R_E]$$

Step 2 – Base Input Impedance



$$Z_b = \frac{V_I}{I_b} \Rightarrow Z_b = r_i + (1 + \beta_D)R_E$$

Since β_D is very large:

$$Z_b \approx \beta_D R_E$$

Step 3 – Source input sees R_B in parallel

The source sees:

$$Z_I = R_B \parallel Z_b$$

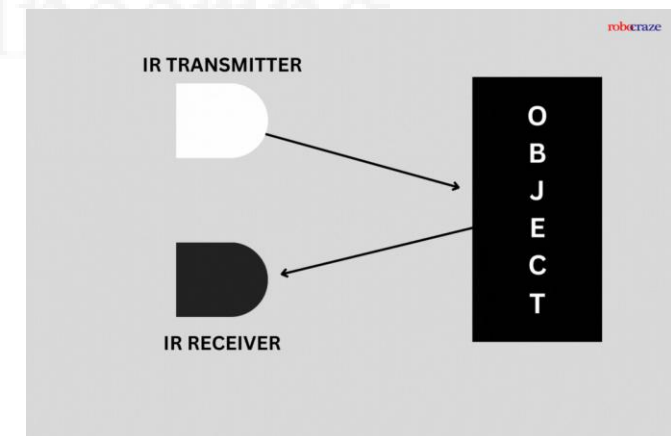
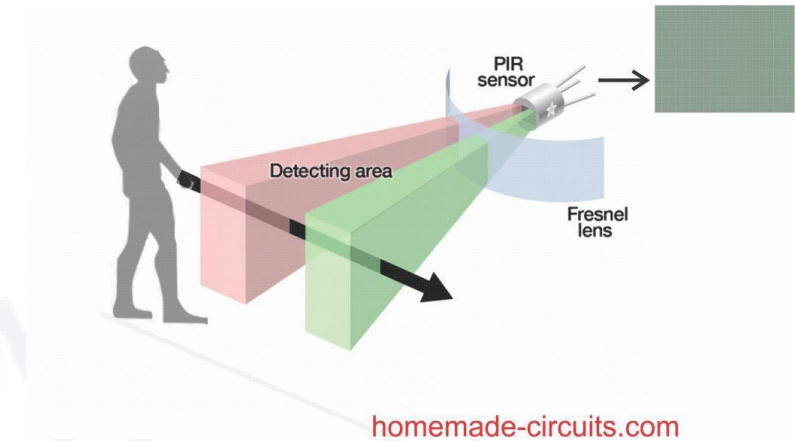
Base impedance is amplified by β_D and R_B appears in parallel $\rightarrow$ Very high input impedance.

Input Impedance Example

Let $\beta_D = 1000$, $R_E = 1\text{ k}\Omega$:

$$Z_b \approx 1000 \times 1\text{ k}\Omega = 1\text{ M}\Omega$$

Interpretation: Very small input current needed → good for sensors.



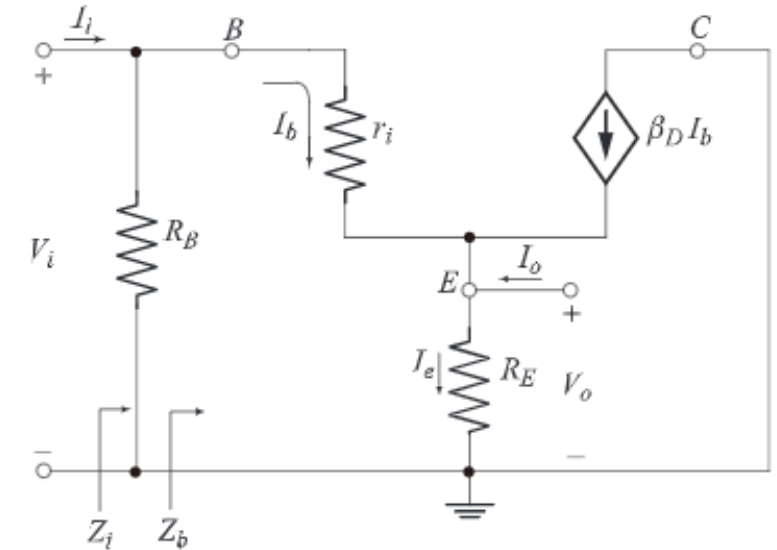
3. AC Current Gain (AI)

Step 1 – Define Current Gain

$$A_I = \frac{I_O}{I_I}$$

Since $I_O = I_e$

$$A_I = \frac{I_e}{I_I}$$



Step 2 – Express Ie

$$I_e = (1 + \beta_D) I_b \approx \beta_D I_b$$

$$A_I = \beta_D \frac{I_b}{I_I}$$

Step 3 – Find I_b/I_I Using KCL

At input node:

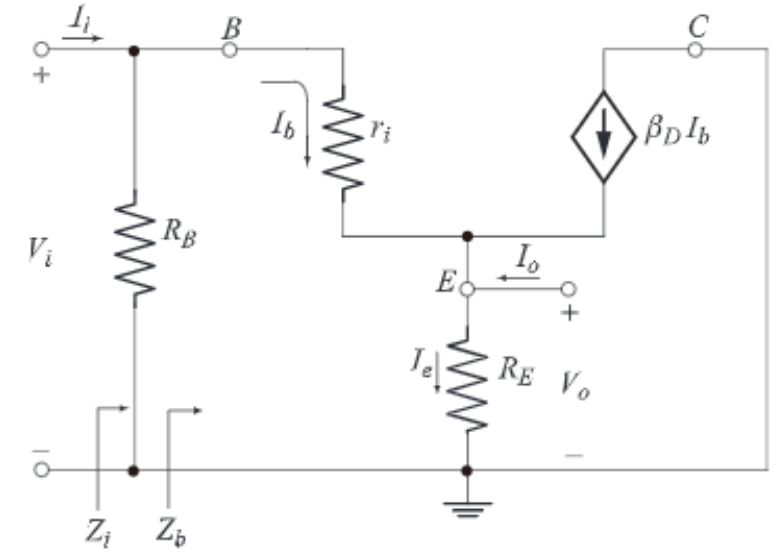
$$I_I = \frac{V_I}{R_B} + I_b$$

But: $V_I = I_b Z_b$

Substitute:

Factor:

$$I_I = \frac{I_b Z_b}{R_B} + I_b = I_b \left(1 + \frac{Z_b}{R_B} \right)$$



$$I_I = I_b \frac{Z_b + R_B}{R_B} \quad \Rightarrow \quad \frac{I_b}{I_I} = \frac{R_B}{Z_b + R_B}$$

Final Current Gain

$$A_I = \beta_D \frac{R_B}{Z_b + R_B}$$

Current gain $\approx \beta_D$ but slightly reduced due to R_B loading.

Current Gain Example

If $\beta = 100$ for each transistor $\rightarrow$ Darlington $\beta_D \approx 100 \times 100 = 10,000$.

With small base current $10\mu A$

$$I_e \approx \beta_D I_b = 10,000 \times 10\mu A = 0.1A$$

Interpretation: Small input drives big load

<https://www.youtube.com/watch?v=a0cZWrm4-y0>

<https://www.instructables.com/Darlington-Pair/>

4. AC Output Impedance (Z_O)

$$Z_O = \frac{V}{I}$$

Step 1 – Apply KCL at emitter

At node **E (emitter)**, the currents entering are I_b through r_i and the dependent current source $\beta_D I_b$. The currents leaving are the emitter current through R_E , call it I_i , and the output current I_o .

$$I_b + \beta_D I_b = I_i + I_o$$

$$I_b + \beta_D I_b - I_i - I_o = 0$$

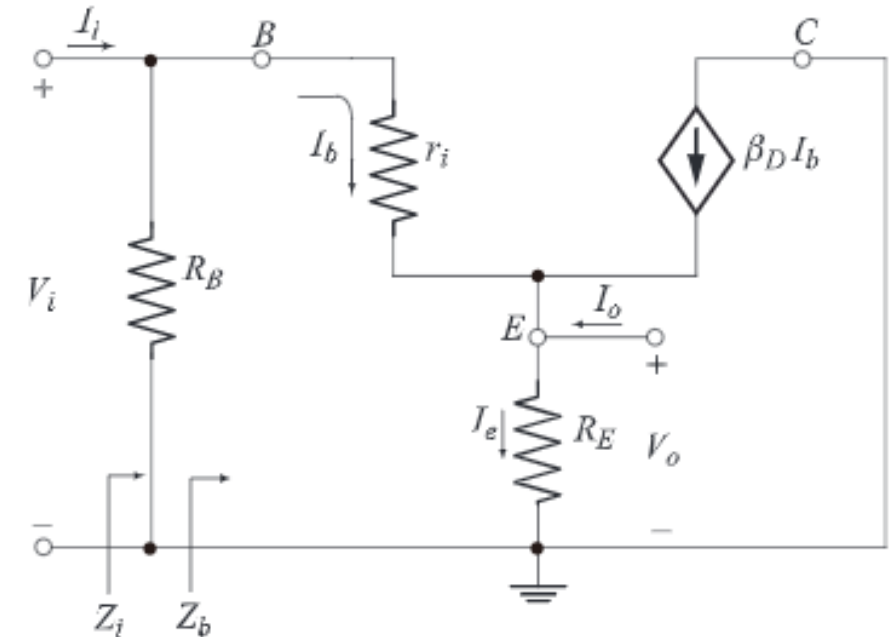
Step 2 — Substitute currents using Ohm's Law

From the circuit:

- The voltage across r_i is **positive at emitter and negative at base**, so the current direction through r_i gives:

$$I_b = -\frac{V}{r_i}$$

Negative sign means current is flowing **opposite** assumed direction)



The current through emitter resistor R_E is:

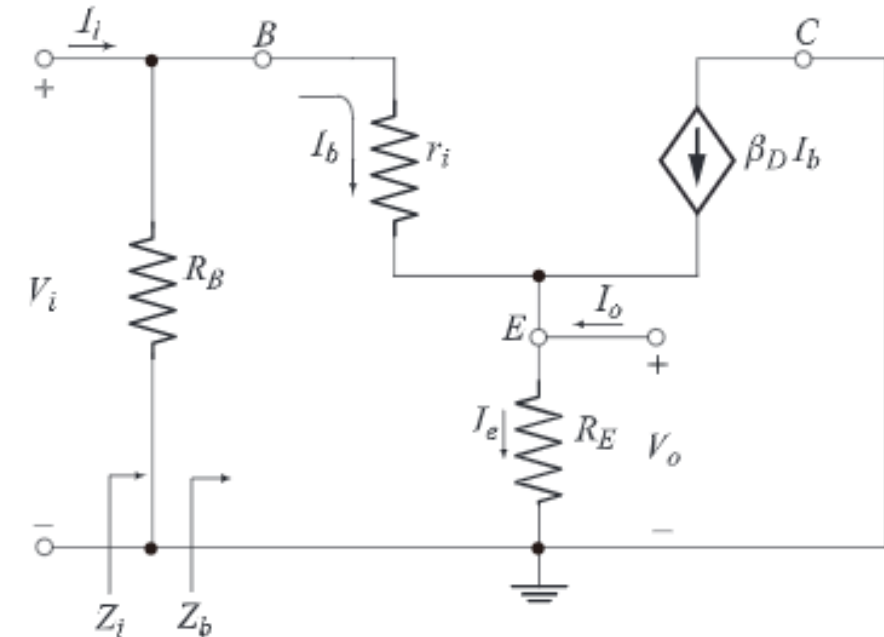
$$I_I = \frac{V}{R_E}$$

Step 3 — Substitute into KCL equation

$$I_b + \beta_D I_b - I_i - I_o = 0$$

$$-\frac{V}{r_i} + \beta_D \left(-\frac{V}{r_i} \right) - \frac{V}{R_E} - I_o = 0$$

$$-\frac{V}{r_i} (1 + \beta_D) - \frac{V}{R_E} - I_o = 0$$



Factor V :

$$I = V \left[\frac{1 + \beta_D}{r_i} + \frac{1}{R_E} \right]$$

Step 2 – Output Impedance

$$Z_O = \frac{V}{I}$$

$$Z_O = \frac{1}{\frac{1}{r_i} + \frac{\beta_D}{r_i} + \frac{1}{R_E}}$$

Rewrite:

$$Z_O = \frac{1}{\frac{1}{r_i} + \frac{1}{r_i/\beta_D} + \frac{1}{R_E}}$$

Recognize these as parallel terms:

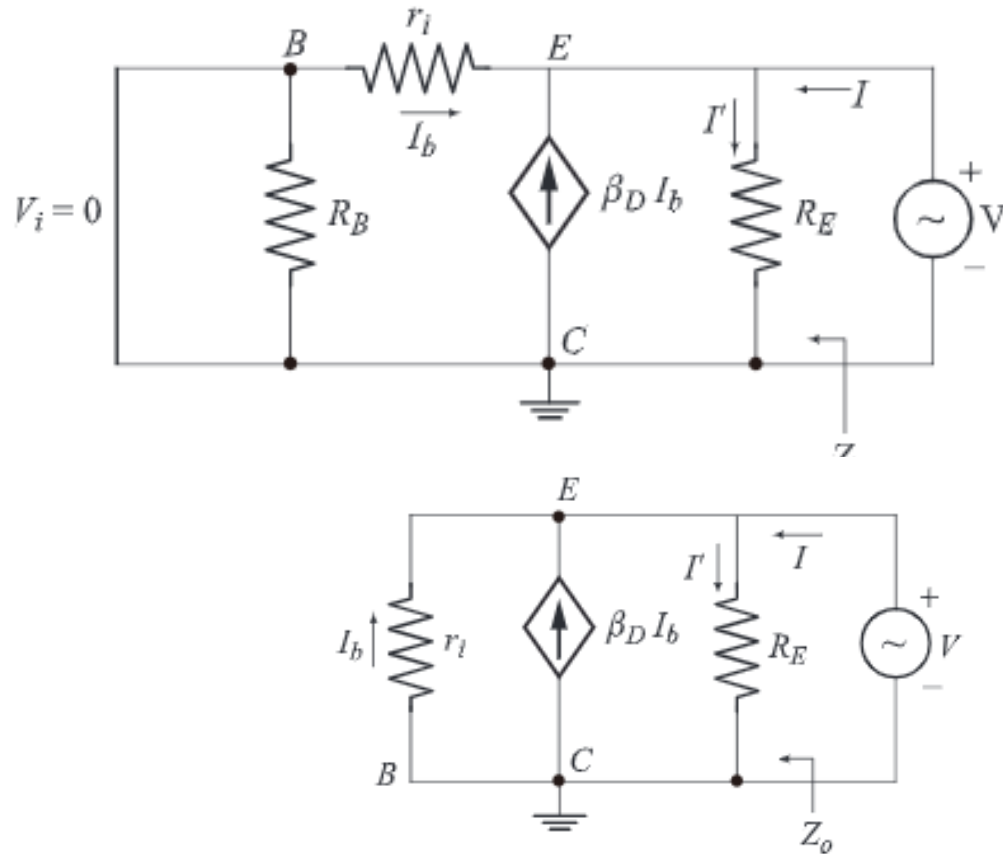
$$Z_O = r_i \parallel R_E \parallel \left(\frac{r_i}{\beta_D} \right)$$

Since r_i/β_D is very small:

$$Z_O \approx \frac{r_i}{\beta_D}$$

On to the leading edge
www.atme.in

Output impedance is very small ($\ll 10 \Omega$) because r_i is divided by β_D .



Output Impedance Example

If $r_i = 50\Omega$ and $\beta_D = 1000$:

$$Z_O \approx \frac{50}{1000} = 0.05\Omega$$

Interpretation: Extremely stiff output → **good for driving loads.**

Why Darlington is useful everywhere)

- ✓ **High input impedance** → Works with tiny sensor signals
- ✓ **High current gain** → Can drive motors, relays, fans
- ✓ **Low output impedance** → Good for power stages
- ✓ **Voltage gain ≈ 1** → Perfect as a buffer stage

DARLINGTON PAIR – FORMULA SUMMARY PAGE

1. AC Voltage Gain (A_v)

$$A_V = \frac{V_O}{V_I} = \frac{(1 + \beta_D)R_E}{r_i + (1 + \beta_D)R_E} \approx 1$$

Meaning: Darlington is a **voltage follower**
→ voltage gain ≈ 1 .

2. Base Input Impedance (Z_b)

$$Z_b = r_i + (1 + \beta_D)R_E \approx \beta_D R_E$$

Meaning: Input impedance becomes **very high**

3. Total Input Impedance (Z_I)

$$Z_I = R_B \parallel Z_b$$

Meaning: Source sees **base impedance in parallel with R_B** .

4. AC Current Gain (A_i)

$$A_I = \beta_D \frac{R_B}{Z_b + R_B}$$

$$A_I \approx \beta_D$$

Meaning: Current is amplified by $\beta \times \beta$ of two BJTs.

5. AC Output Impedance (Z_o)

$$Z_O = r_i \parallel R_E \parallel \left(\frac{r_i}{\beta_D} \right)$$

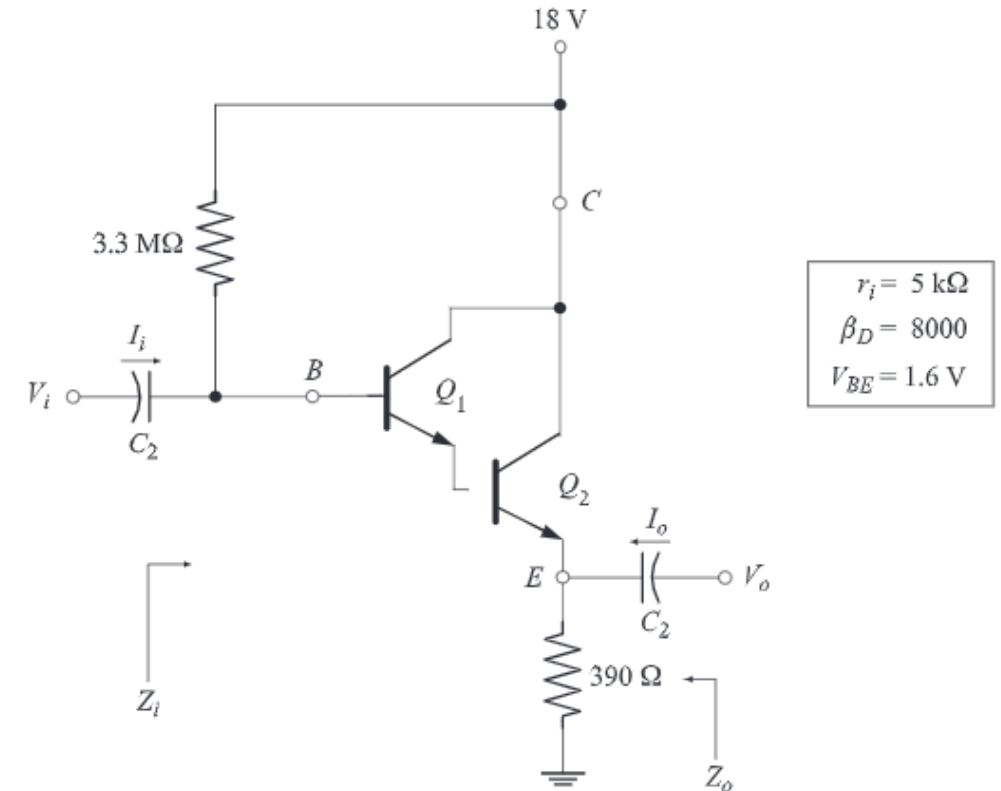
$$Z_O \approx \frac{r_i}{\beta_D}$$

Meaning: Output impedance becomes **very low**.

Numerical: Darlington Emitter Follower

For the Darlington emitter–follower shown below:

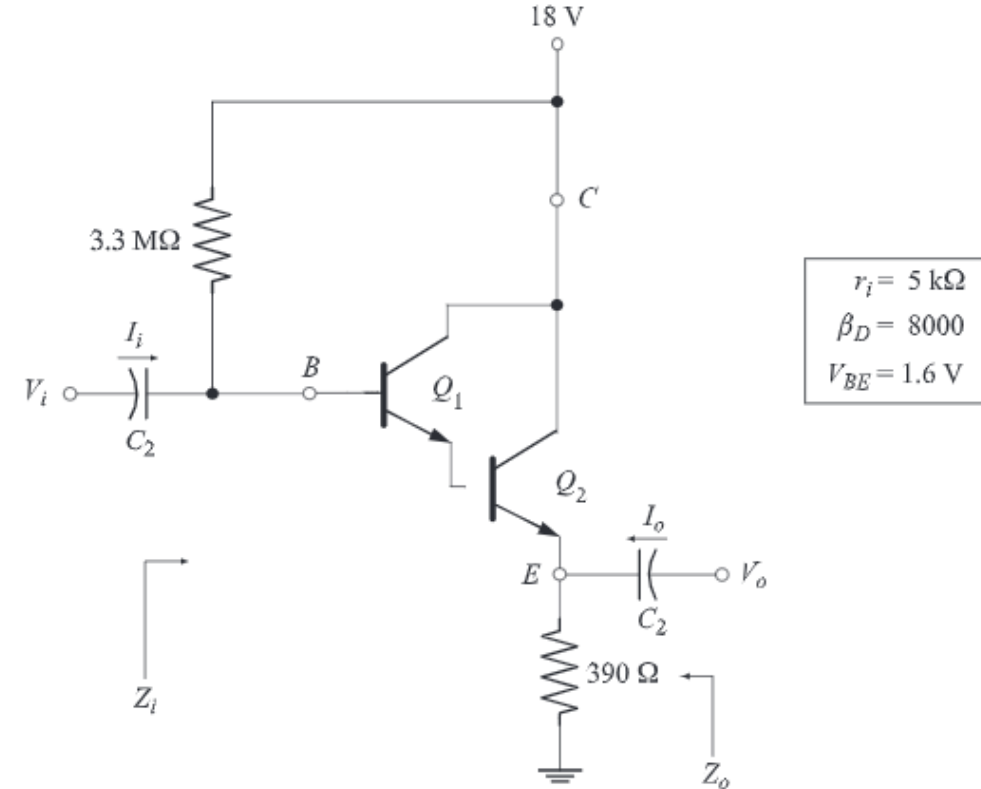
- Calculate the dc bias voltages V_B , V_E , V_C and currents I_B and I_C .
- Calculate the input and output impedances.
- Determine the voltage and current gains.



- Supply: $V_{CC} = 18\text{ V}$
- Base resistor: $R_B = 3.3\text{ M}\Omega$
- Emitter resistor: $R_E = 390\text{ }\Omega$
- Composite current gain of Darlington: $\beta_D = 8000$
- Two base-emitter junctions in series $\rightarrow$ total base-emitter drop
 $V_{BE} \approx 0.8 + 0.8 = 1.6\text{ V}$

Starting at V_{CC} and going through R_B , the base-emitter junctions, and R_E to ground:

$$V_{CC} = I_B R_B + V_{BE} + I_E R_E$$



For a Darlington, the emitter current is approximately

$$I_E \approx \beta_D I_B$$

$$V_{CC} = I_B R_B + V_{BE} + (\beta_D I_B) R_E$$

Group the terms with I_B :

$$V_{CC} - V_{BE} = I_B (R_B + \beta_D R_E)$$



$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E}$$

Step-1:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E}$$

$$I_B = \frac{18 \text{ V} - 1.6 \text{ V}}{3.3 \text{ M}\Omega + (8000)(390 \text{ }\Omega)}$$
$$= \frac{16.4}{6.42 \times 10^6} \approx 2.55 \times 10^{-6} \text{ A}$$

$$I_B \approx 2.55 \text{ }\mu\text{A}$$

Step-2:

$$I_E = I_{E2} \approx I_{C2} = \beta_D I_B$$

$$I_E = \beta_D I_B$$

$$I_E = 8000 \times 2.55 \text{ }\mu\text{A}$$
$$= 20.4 \text{ mA}$$

Since $I_C \approx I_E$ for a large- β transistor, we also have

$$I_C \approx 20.4 \text{ mA}$$

Step-3:

Finding the emitter voltage V_E

Emitter resistor drop:

$$\begin{aligned} V_E &= I_E R_E \\ &= (20.4 \text{ mA})(390 \Omega) \end{aligned}$$

$$V_E = 0.0204 \times 390 \approx 7.96 \text{ V}$$

$$V_E \approx 7.96 \text{ V}$$

Step-4:

Finding the base voltage V_B

The base is above the emitter by the total Darlington base–emitter voltage:

$$\begin{aligned} V_B &= V_{BE} + V_E \\ &= 1.6 \text{ V} + 7.96 \text{ V} = 9.56 \text{ V} \end{aligned}$$

$$V_B \approx 9.56 \text{ V}$$

Step-5:

Finding the collector voltage V_C

In this circuit, the collectors of the transistors are tied directly to V_{CC} .
Therefore the collector DC voltage is simply:

$$V_C = V_{CC} = 18 \text{ V}$$

Step-6:

Impedance seen **looking into the base** is

$$Z_b = \frac{v_b}{i_b} = r_i + (1 + \beta_D) R_E$$

$$Z_b = 5 \text{ k}\Omega + (8001)(390 \Omega)$$

$$8001 \times 390 \approx 3.12 \times 10^6 \Omega = 3.12 \text{ M}\Omega$$

Add the 5 k Ω :

$$Z_b \approx 3.13 \text{ M}\Omega$$

Step-7: Total **input** impedance $\rightarrow Z_i$

$$Z_i = R_B \parallel Z_b = 3.3 \text{ M}\Omega \parallel 3.13 \text{ M}\Omega \\ = 1.6 \text{ M}\Omega$$

Why parallel?

At the base node, for AC:

- R_B is from base to ground.
- The transistor input is Z_b from base to ground.

Parallel formula:

$$Z_i = \frac{R_B Z_b}{R_B + Z_b} \quad Z_i = \frac{(3.3)(3.13)}{3.3 + 3.13} \text{ M}\Omega \approx \frac{10.329}{6.43} \text{ M}\Omega \approx 1.6 \text{ M}\Omega$$

Step-8

$$Z_o = r_i \parallel R_E \parallel \frac{r_i}{\beta_D}$$

$$Z_o = 5 \text{ k}\Omega \parallel 390 \Omega \parallel \frac{5 \text{ k}\Omega}{8000} = 0.625 \Omega$$

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Step-9

$$A_V = v_o / v_i$$

$$A_V = \frac{v_o}{v_i} = \frac{(1 + \beta_D) i_b R_E}{i_b [r_i + (1 + \beta_D) R_E]} = \frac{R_E (1 + \beta_D)}{r_i + R_E (1 + \beta_D)}$$

$$A_V = \frac{(390 \, \Omega)(8001)}{5 \, \text{k}\Omega + (390 \, \Omega)(8001)}$$

$$A_V = \frac{3 \, 120 \, 390}{3 \, 125 \, 390} \approx 0.998$$

$$A_V \approx 0.998$$

Step-10

$$A_I = I_o / I_i$$

$$A_I \approx \frac{\beta_D R_B}{R_B + \beta_D R_E}$$

$$A_I = \frac{(8000)(3.3 \text{ M}\Omega)}{3.3 \text{ M}\Omega + (8000)(390 \Omega)}$$

$$A_I \approx 4.1 \times 10^3$$

Step-11

$$V_o = A_v V_i = (0.998)(120 \text{ mV})$$
$$= 119.76 \text{ mV}$$